Foundation of Base-Level Activation in the Environment

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- Base-level activation in ACT-R is supposed to reflect the log odds that a memory is needed now given its past history.
- Based on studies of repetition of items in the environment (Anderson & Schooler, 1991, and much subsequent work of Schooler) the base-level action of memory $i$ is:
  \[ B_i = \log(\sum_{j=1}^{n} t_j^{-d}) + B \]
  where $t_j$ is time since $j$th presentation of $i$.
- This model accounts for the effects of retention interval and practice and some of the interactions of the two, both in the environment and in human memory.
- However, it does not account for the spacing effect, which is found in both the environment and memory -- Pavlik’s (Pavlik & Anderson, 2005) extension to ACT-R does.
Today we have access to much larger data bases and the compute power to process them, enabling us to perform a much more detailed analysis.

- **Data Base 1 (Stanley, 2014)**: All tweets from the top 500 most followed English tweeters as of Jan 7, 2014, from July 11, 2007 to Jan 7, 2014.
- **Reddit Data Base**: The top 500 subreddits (by subscribers) on two days, April 23 and May 5, 2021. The visible comments in the top 25 topics.

### Analysis of the New Data Bases

- **Twitter**: Following a tweeter what is the probability of a string in a message like *This debate is not just about numbers. It's a set of major decisions that are going to affect millions of families.*
- **Reddit**: Reading the comments on a subreddit what is the probability of a string in a message like *I received 100% attendance in 4th grade and got a free ice cream dessert at Ponderosa.*
- **The Question**: Focusing on 20,000 most frequent non-functor strings, what is the probability of a string in the next message (tweet from a tweeter or comment on a subreddit) given its pattern of occurrence over the last 1000 messages.
- **Twitter**: Over a million tweets, averaging 7.5 unique strings, offering over a billion 1000-tweet patterns.
- **Reddit**: Over a million comments, averaging 14.9 unique strings, offering over a billion 1000-comment patterns.
Patterns of Repetition

- Prob^~.0255*Lag^-5.4*N^-6.5* R^2=.843 in log scale.
- Curves are negatively accelerated to a point after which they become positively accelerated.
- The “final crash” reflects a response to massing of presentations a long time ago.
- Short lags are best at brief delays and worst at long delays.
- Short lags at long delays not much better than a single presentation.
ACT-R without Spacing

(a) Combined Data: Recency and Frequency

(b) Combined Data: Spacing Effects

Pavlik Model of Spacing

(a) Combined Data: Recency and Frequency

(b) Combined Data: Spacing Effects

(a) Pavlik Model: Recency and Frequency

(b) Pavlik Model: Spacing Effects

$B_i = \ln\left(\sum_{j=1}^{n} t_j^{-d_i}\right)$; $d_i = d$; $c = 0.456$; $d = 0.775$
Anderson & Milson (1989) Model

1. The desirabilities, λ’s, of items are distributed according to a gamma function: \( \pi(\lambda) = \frac{\lambda^{v-1}e^{-\lambda/b}}{\Gamma(v)b^v} \), where \( \Gamma(v) = (v-1)! \) for integer \( v \).

2. Items decay in desirability over time according to an exponential function: \( r(t) = e^{-dt} \). (I will also explore the power function \( r(t) = t^{-d} \).)

The above 2 assumptions are derived from Burrell’s model of library borrowing and imply that the expected probability of an item occurring that has had \( n \) occurrences in time \( t \) is

\[
F(t) = e^{-M(t)/b}
\]

where \( M(t) = \int_0^t r(s)ds \).

3. The rate of decay, \( d \), varies for items: \( f(d) = e^{-d/\alpha} \).

4. With each unit of time there is a probability \( p \) that the item will have a revival of desirability to original level.

Assumptions 3 & 4 eliminate the possibility of a closed form calculation and require simulation.

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### Original Anderson & Milson Model

<table>
<thead>
<tr>
<th>(a) Combined Data: Recency and Frequency</th>
<th>(b) Combined Data: Spacing Effects</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Gamma Params:</strong> v = .164, b = .139</td>
<td>Mean Rate = .023</td>
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<td>Probability of a revival ( p = .003; ) Mean time to Revival = 333 messages</td>
</tr>
<tr>
<td>Exponential Distribution of ( d ) in ( e^{-d/\alpha} ), ( \alpha = .035 ); average of 22% left after 100 messages</td>
<td>( R^2 = .904 )</td>
</tr>
</tbody>
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### Exponential Decay: Recency and Frequency

- N=197-225
- N=170-196
- N=145-169
- N=122-144
- N=101-121
- N=82-100
- N=65-81
- N=50-64
- N=37-49
- N=26-36
- N=17-25
- N=10-16
- N=5-9
- N=2-4
- N=1

### Exponential Decay: Lag Effects

- Lag=1
- Lag=2-9
- Lag=10-49
- Lag=50-225
- Lag>225

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### Probability in Next Message (log Scale)

- Number of Intervening Messages (Log Scale)
- 0.0005
- 0.005
- 0.05
- 0.5

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### Number of Intervening Messages (Log Scale)

- 1
- 2
- 5
- 10
- 25
- 100
- 250
- 1000
Anderson & Milson: Power Decay

(a) Combined Data: Recency and Frequency

(b) Combined Data: Spacing Effects

Gamma Params: \( v = .199, b=.482 \). Mean Rate = .096
Exponential Distribution of \( d \) in \( t^{-d} \), \( \alpha = 4.08 \); average of 5% left after 100 messages
Probability of a revival \( p = .001 \); Mean time to Revival =1000 messages
\( R^2 = .957 \)

A Mechanism for Anderson & Milson

- A&M’s Monte Carlo simulation is implausible as model of human memory and not something to run as part of ACT-R
- However, it suggests something like to PPE (Walsh, Gluck, Gunzelmann, Jastrzembski, Krusmark (2018):

\[ B_i = \pi_i \cdot T_i^{-d_i} \]

- \( T_i \) is an estimate of the time since the last revival estimated as the harmonic of prior \( t_p \) plus observed times \( t_1, t_2, ..., t_n \).
- The initial desirability and the decay are functions of the effective interval \( M=(G_p +G)/2 \) where \( G = (t_n-t_1)+1 \):

\[ \pi_i = \frac{a \cdot n}{M}, \quad d_i = \frac{b}{M} \]

- Note opposing effects of \( M \) and hence gap \( G \).
- Note that with respect to decay the critical feature is not the spacing of interval between the items but the period \( G \) over which they appear.
Conclusions

- Assuming power-law decay Anderson & Milson is a surprisingly good model of the memory demands of the environment.
- A simple PPE-like equation can be used which is easy to compute and captures spacing effects: \( B_i = \pi_i \times T_i^{-d_i} \).
- This equation suggests that the critical variable might not be the spacing between individual items but rather the period of time spanned by all presentations.
- While high-density presentation has not been a focus of memory research, it happens in the real world and it is implausible that all those close presentations (e.g. of my wife’s name) are causing rapid forgetting.
- Preliminary efforts to extend \( B_i = \pi_i \times T_i^{-d_i} \) to human memory experiments appear to do a good job using the ACT-R

\[
prob = \frac{1}{\frac{\tau - B_i}{1 + e^{-s}}}
\]