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Learning From Examples Versus Verbal Directions in Mathematical Problem Solving

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ABSTRACT— This event-related fMRI study investigated the differences between learning from examples and learning from verbal directions in mathematical problem solving and how these instruction types affect the activity of relevant brain regions during instruction and solution periods within problem-solving trials. We identified distinct neural signatures during the instruction period of trials. While studying examples, greater activation was found in the prefrontal and parietal regions that were known to be involved in mathematical problem solving. In contrast, while studying verbal directions, increased activation was found in motor and visual regions. These differences, however, disappeared during the solution period. During the solution period, participants showed brain activation patterns like those they displayed while studying an example, regardless of which instruction they learned from. The results suggest instruction type becomes irrelevant after students get to an understanding. Educational implications were discussed with regard to example-based instruction.

When learning to solve a problem, we process some form of instruction and transform our understanding of the instruction into actual problem-solving behavior. Instructional information often takes two alternative forms, an example or verbal direction. Examples illustrate how a procedure is applied to a problem and has a similar appearance to the task items. In contrast, verbal directions usually consist of a series of sentences that describe how the task should be done. Many teachers and students use various combinations of these two methods. Despite their pervasiveness, however, the neural mechanisms by which we learn to perform new tasks from examples versus verbal directions are still poorly understood. If we had a better understanding, we might be able to better select the appropriate instructional combination for different learning goals. In this article, we want to examine the effect of these two alternative forms of instruction on learning in mathematical problem solving and identify how these two methods affect the activity of relevant brain regions.

Learning From Examples Versus Verbal Directions

Comparing learning from examples with learning from verbal directions, each form of instruction seems to have its particular advantages and disadvantages. The biggest advantage of an example is that it illustrates how the procedures are applied. Because of similar appearance between an example and a problem, students often prefer to learn from examples rather than written instructions (LeFevre & Dixon, 1986; Pirolli & Anderson, 1985). However, learning from examples is not always successful. Learners may infer an incorrect rule while studying an example (Matsuda, Lee, Cohen, & Koedinger, 2009). Also, they may not be able to solve problems that are slightly different from the studied examples (Reed, Dempster, & Ettinger, 1985).

In contrast, verbal directions describe the steps for solving a problem. Written instructions can specify where to look, what to do, and how to respond. However, such procedures can be abstract and learners may have difficulty comprehending them. Also, because verbal directions are not similar in appearance to the task items, unlike an example, students have to mentally translate written instructions into an executable form so that they can create a specific instance of the problem (Anderson, Farrell, & Sauers, 1984).

Although both examples and verbal directions are common instructional methods for problem solving, we do not have a good understanding of the mental processes by which they are comprehended and how they are different from each

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other. There are relatively few behavioral studies that directly compare use of examples and verbal directions. Reed and Bolstad (1991) compared the effects of providing an example with providing a set of verbal procedures and showed that providing an example was more effective. On the other hand, other studies have shown that verbal instructions and examples were equally effective (e.g., Cheng, Holyoak, Nisbett, & Oliver, 1986; Fong, Krantz, & Nisbett, 1986). For instance, Fong et al. (1986) showed that students who were trained with explicit rules performed as well as those who were trained with specific examples in learning of the law of large numbers. Finally, in some of our past research on learning complex problem solving, we have found that some students were unable to learn from examples while they could successfully learn when given verbal directions (Lee, Fincham, Betts, & Anderson, 2014).

Given mixed results about the relative efficacy of examples and verbal directions, we suspect that which of these instructional methods is more effective depends on the task and the quality of instruction. Good examples are useful, but not all examples are equally effective (Lee & Anderson, 2013). Likewise, what has been emphasized during instruction could lead to different learning outcomes (e.g., Perry, 1991; Rittle-Johnson & Alibali, 1999). We believe the mixed results regarding different instructional methods suggest that we do not have a good understanding about the underlying mechanisms by which instructional method influences learning. Thus, our goal in this study is not to simply pit one instructional method against another. Rather, we will investigate a situation where both methods are relatively successful and look at the different neural signatures of the learning that results. Examining these brain level effects might help one understand how to select a combination of the instructional methods to achieve one's learning goals.

Neural Signatures Between Learning From Examples Versus Verbal Directions

To our knowledge, no study has contrasted the distinct neural signatures between these two alternative forms of instruction. Most previous studies have focused on identifying relevant brain regions underlying execution of instructed tasks without distinguishing different types of instructions (Brass, Wenke, Spengler, & Waszak, 2009; Cole, Bagic, Kass, & Schneider, 2010; Hartstra, Kühn, Verguts, & Brass, 2011; Ruge & Wolfensteller, 2010; Stocco, Lebiere, O'Reilly, & Anderson, 2010; Stocco, Lebiere, O'Reilly, & Anderson, 2012). In such prior research, participants are presented with instructions on the screen and then they have to apply these instructions to the problems that follow. Such paradigms mostly have focused on distinguishing the period when the instruction is studied from the period when it is applied. Several studies have reported that lateral prefrontal and posterior parietal regions are engaged during encoding of instructions for new tasks and execution of novel instructions (e.g., Brass et al., 2009; Cole et al., 2010; Hartstra et al., 2011; Ruge & Wolfensteller, 2010).

In the current study, we want to examine whether there are neural signatures that distinguish learning from examples and learning from verbal directions in a mathematical problem-solving task. In addressing this question we need to distinguish effects of the two types of instruction in the study period versus the solution period. We want to answer the following questions: (1) what regions are more engaged in studying of an example and what regions are more engaged in studying of verbal directions? (2) If both methods result in successful learning are there any differences when participants apply their knowledge in problem solving? and (3) which type of learning is more similar to actual problem-solving behavior?

We expect that when learners study a mathematical example they will show activation in regions that are pertinent to mathematical reasoning. The brain activity should be similar to activity when actually solving a problem because both study of examples and problem-solving require mathematical computation. In contrast, while studying verbal directions (vs. examples), learners would exhibit greater activation in the regions that are pertinent to reading activity. Learners have to read and memorize written instructions and this will cause greater activity in regions that are responsible for processing visual information. Such reading activity will not necessarily require mathematical computation, thus we expect that studying verbal directions will be distinctively different from actual problem-solving behavior.

Although we expect different patterns of brain activity between the two different types of instruction (example vs. verbal direction) during the study period, we do not expect this to translate to different brain activity during the solution period. When learners succeed in figuring out problem-solving rules either from examples or verbal directions, we expect they will create the same internal representation that connects what features of the task to pay attention to, what operators to perform, and how to respond. We believe this is because such internal representation does not have to depend on the form of initial instruction. In a prior study (Lee et al., 2014), contrasting discovery learning versus instruction (combined example and verbal directions), we did not find brain activation differences after the skill had been mastered. Similarly, Klahr and Nigam (2004) also demonstrated behaviorally that what is learned was more important than how it is learned in the domain of science education. The hypothesis that instruction type does not matter after getting to an understanding predicts an interaction effect between instruction type and period such that learning from example versus verbal direction will cause different brain activity during the study period but this difference will disappear during the solution period. The alternative hypothesis is that learners will develop different internal representations depending on the forms of provided instruction and this will in turn affect problem-solving activity while applying their representation to the actual task item. This hypothesis predicts that we should still see differences in problem solving, perhaps like the differences expected during the instruction period.

According to our hypotheses, brain regions that are pertinent to mathematical problem solving are expected to show an interaction between instruction type (example vs. verbal direction) and period type (study vs. solution) because brain activity should differ only in the study period, but not in the solution period. More specifically, these regions should show lower activity when studying verbal directions than when studying an example or solving a problem (given either form of instruction). This is because only during the study phase of verbal directions, mathematical computation is not required whereas during all other conditions mathematical computation is required.

In particular, we expected such an interaction in two regions (coordinates are given in Table 1). One is the horizontal intraparietal sulcus (HIPS) which past research has shown to be engaged by mathematical tasks and the processing of quantity (e.g., Cohen-Kadosh, Lammertyn, & Izard, 2008; Dehaene, 1997; Dehaene & Cohen, 1997; Dehaene, Piazza, Pinel, & Cohen, 2003; Lee et al., 2014; Rosenberg-Lee, Lovett, & Anderson, 2009). For instance, we (Lee et al., 2014) have shown that while people are solving problems that require algebraic transformation, activity in the HIPS regions increased relative to when solving non-algebraic problems. Consistently, Rosenberg-Lee et al. (2009) showed activity of the HIPS increased with task difficulty in a multi-digit multiplication task.

The other region that we expected to show an instruction by period interaction is the lateral inferior prefrontal cortex (LIPFC) which past research has demonstrated to be involved in the retrieval of arithmetic facts (Danker & Anderson, 2007; Menon, Rivera, White, Glover, & Reiss, 2000). This region is also shown to be involved in the retrieval of semantic facts more generally (e.g., Thompson-Schill, D'Esposito, Aguirre, & Farah, 1997; Wagner, Paré-Blagoev, Clark, & Poldrack, 2001). Besides showing an interaction, we expected that both the LIPFC and HIPS would be more active during problem solving than instruction due to their critical role in mathematical reasoning.

We also chose to focus on the fusiform area, which we thought would be more active during instruction study than during the solution period. The fusiform area is known for visual processing of attended information (Anderson, 2007). Several studies have shown that the fusiform area plays an important role in visual word recognition (Price & Mechelli, 2005; Leff et al., 2001; Shaywitz et al., 2002; Turkeltaub, Gareau, Flowers, Zeffiro, & Eden, 2003). Other research has shown that the fusiform has been implicated in perceptual recognition (Grill-Spector, Knouf, & Kanwisher, 2004; McCandliss, Cohen, & Dehaene, 2003) and mental imagery (D'Esposito et al., 1997; Ishai, Ungerleider, & Haxby, 2000). All of these prior studies highlight the importance of the fusiform area in visual processing of attended information. Therefore, we expect this region will be engaged by detailed visual processing that would be evoked by study of either verbal instruction or example. While it is still necessary to encode information from the problem during the solution period, the majority of the effort should be focused on the actual calculation of an answer. Thus, we predicted the fusiform would be more active during study than problem solving.

The final region we chose for potential interest was the angular gyrus (AG). Numerous studies have suggested the AG is involved in mathematical cognition. According to the triple-code theory (Dehaene et al., 2003), the left AG is involved in verbal processing of numbers as part of a left lateralized perisylvian language network. We (Lee et al., 2014) observed that people tended to do verbalization to help their computation and reduce working memory load and this in turn tended to increase the activity in the AG area. The AG is also known to be involved in the retrieval of declarative arithmetic facts (Dehaene et al., 2003; Grabner et al., 2009; Schmidthorst & Brown, 2004). According to these previous studies, we can predict that the AG area will show greater activation during the solution period when people have to do mathematical computation. On the other hand, some other research suggests the AG plays a metacognitive role in mathematical problem solving. For example, in both Anderson, Betts, Ferris, and Fincham's (2011) and Wintermute, Betts, Ferris, Fincham, and Anderson's (2012) studies, the AG appeared to play a metacognitive role when people had to extend their acquired knowledge to a new mathematical task. These results predict that the AG will be more involved during instruction study than during the solution period because in our experiment people have to infer a problem-solving rule while they are studying the instruction. In contrast, they can simply apply the rule they have inferred when they are solving a test problem. Other research also suggests the AG is involved in a wide range of semantic tasks (e.g., Binder, Desai, Graves, & Conant, 2009; Desai, Binder, Conant, Mano, & Seidenberg, 2011; Rapp, Mutschler, & Erb, 2012; Seghier, Fagan, & Price, 2010) and visuospatial processing (Cattaneo, Silvanto, Pascual-Leone, & Battelli, 2009; Göbel, Walsh, & Rushworth, 2001). Again this would suggest a greater role during study. Given the conflicting prior research, this is the one region for which we did not have a prediction of whether it would be more active during study or solution. By including this region, we hoped to perhaps

Table 1
Locations of Predefined Regions

ROI	*Brodmann area(s)*	*Volume*	*Talairach coordinates (x, y, z)*
HIPS	40	12.8 mm (high) by $12.5 \times 12.5\ \text{mm}^2$	−34, −49, 45
LIPFC	9 and 46	12.8 mm (high) by $15.6 \times 15.6\ \text{mm}^2$	−43, 23, 24
AG	39	12.8 mm (high) by $12.5 \times 12.5\ \text{mm}^2$	−41, −65, 37
Fusiform	37	9.6 mm (high) by $12.5 \times 12.5\ \text{mm}^2$	−42, −61, −9

Note: The coordinates were chosen based on prior studies: LIPFC and Fusiform from the ACT-R theory (Anderson, 2007) and HIPS and AG from Dehaene et al. (2003) and Kadosh et al. (2008).

shed more light on the function of the AG and its role in mathematical cognition.

Because of their importance in past research, we chose our predefined regions described above from the following prior studies: the LIPFC and fusiform from the ACT-R theory (Anderson, 2007) and the HIPS and AG from Dehaene et al. (2003) and Kadosh, Lammertyn, and Izard (2008). Table 1 shows coordinates and dimensions of these predefined regions of interest. For all predefined regions, we focused on the left hemisphere to minimize the number of pairwise comparisons, and also because the left hemisphere has generally shown a stronger effect than the right hemisphere for mathematical problem-solving tasks in past studies (e.g., Anderson et al., 2011; Lee et al., 2014; Wintermute et al., 2012)

CURRENT EXPERIMENT

The current study focused on the effects of two alternative forms of instructions—examples versus verbal directions when participants learned to solve simple mathematical problems. To collect multiple observations during the experiment, we introduced a new "task" on every trial and participants had to learn the new task either from an example or from verbal direction. Participants therefore studied an example or verbal direction presented on the screen and then applied the rule they inferred to a problem that was presented on the following screen. In this way, we were able to examine the effect of two different forms of instruction on each of the study period and the solution period.

METHOD

Participants

Twenty graduate and undergraduate students (14 male and 6 female, $M = 23.9$ years, $SD = 3.5$) from Carnegie Mellon University participated in this study. All participants were right-handed and had normal or corrected-to-normal vision. Participants received $60 plus a performance-based bonus (5 cents per correctly solved problem).

Learning Materials

We developed a series of mathematical problems that required a simple arithmetic computation. This task was developed based on some of our earlier research (Anderson, Lee, & Fincham, 2014; Lee et al., 2014; Lee, Betts, & Anderson, 2015) where we studied students learning an extensive curricula of "data-flow" diagrams that required filling in the empty boxes in a diagram. In the current task, we used a simplified task so that we can focus on the differences between learning from examples versus verbal directions.

Figure 1 illustrates instructions provided in the example and verbal direction conditions for a particular problem used in this study. In both conditions, both an example and a verbal direction were presented together, but, depending on the condition, only one type of information was useful for inferring the rule. In the example condition (see Figure 1a) participants had to determine where the number in the blue box (i.e., 13) comes from by identifying two numbers and one operator that are relevant for computation. In this particular case, 13 can be obtained by applying the subtraction operator (−) from the top box to the numbers in the top box. Note of course that every problem would involve selecting a different combination of operators and numbers as described below. In contrast, in the verbal direction condition (see Figure 1b), the instruction explicitly describes the rule for solving the upcoming problem. Note there are verbal instructions in the example condition and similarly an example in the verbal direction condition. However, these are purposely vague and provide no information as to how to solve the upcoming problem, but simply serve to keep the amount of visual information constant between the two conditions.

After studying the instruction during a trial, participants were presented with a problem like that shown in Figure 1c. Here, participants have to determine a number to fill in the empty box given the three numbers and two operators provided. Notice that if given just the problem alone there are a number of possible solutions. It is the instruction studied just before that determines the correct rule to apply. In this example, the correct answer is 15, because $11 + 4 = 15$.

Figure 1 is just one instantiation of the problem-solving rules we used. The rule was changed for every trial by changing the location of numbers and operators required for the

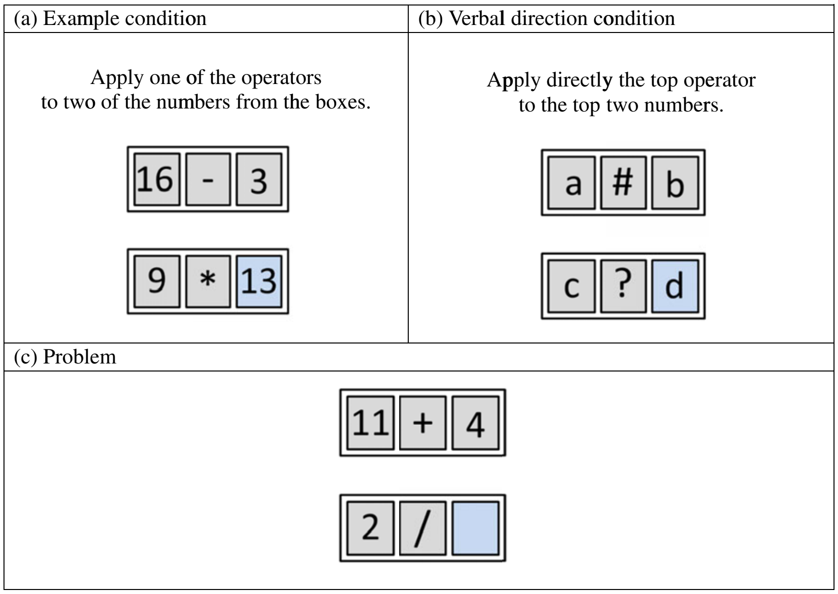

Fig. 1. An example of instructional information in (a) example condition and (b) verbal direction condition. By applying the same rule presented in either (a) or (b), participants can solve a subsequent problem as shown in (c). The correct answer to this problem is 15 because $11+4=15$.

computation of the answers. For all of the problems, however, the rule always involved two numbers and one operator to keep the complexity of the problem constant across the conditions. For instance, another rule might be: "Apply the inverse of the bottom operator to the left two numbers." When applying this rule to the problem shown in Figure 1c, the correct answer is 22 because $11*2=22$. Because our main research interest is to identify brain signatures that are distinct between two alternative forms of instruction, the problems were constructed such that each could be solved by our participants given either type of instruction. We wanted to achieve similar levels of high accuracy for both conditions so that we could focus on how the relevant knowledge is acquired and later used to solve problems.

Design

A within-subjects, event-related design was employed to test the effect of example versus verbal direction while learning a type of mathematical problem solving. All participants were exposed to both example-instruction (Example) and verbal-instruction (Verbal) trials. These two trial types were alternated throughout the experiment while the specific ordering was counterbalanced across the participants.

During example trials, the helpful example changed according to the problem-solving rule for each trial, but the same unhelpful written instruction (as shown in Figure 1a) was used repeatedly. During verbal trials, the helpful written instruction changed according to the problem-solving rule for each trial, but the same unhelpful example (as shown in Figure 1b) was used repeatedly. The helpful verbal directions were created by appropriately filling in the template: "Apply [directly/inverse of] the [top/bottom] operator to the [top/bottom/left/right/diagonal] two numbers."

By varying the location of the answer box (blue tile in Figure 1), we constructed four different types of problems (top left, top right, bottom left, and bottom right). Within each of the four types, we varied the features of the instructional information to construct 12 different problem-solving rules. We varied the type of operator (given vs. inverse), location of operator (top vs. bottom) and location of two numbers (top/bottom two numbers vs. left/right two numbers vs. diagonal two numbers). These 12 rules ($2\times2\times3=12$) were instantiated as both example trials and verbal trials resulting in a total of 24 different problems. Four of these 24 problem sets were constructed. Within any particular set, the order of problems was randomized and the same rule never appeared consecutively when alternating the example and verbal direction trials. In both examples and problems, the top operator and bottom operator were never identical. The answer for any problem was always a two-digit number.

Procedure

As described above, experimental trials were grouped into four sets, with each set consisting of 24 trials, alternating type (Example, Verbal or vice versa) from trial to trial. The first problem set was meant for practice to become familiarized with the task, and as such data from this set were not analyzed. Participants solved this practice problem set

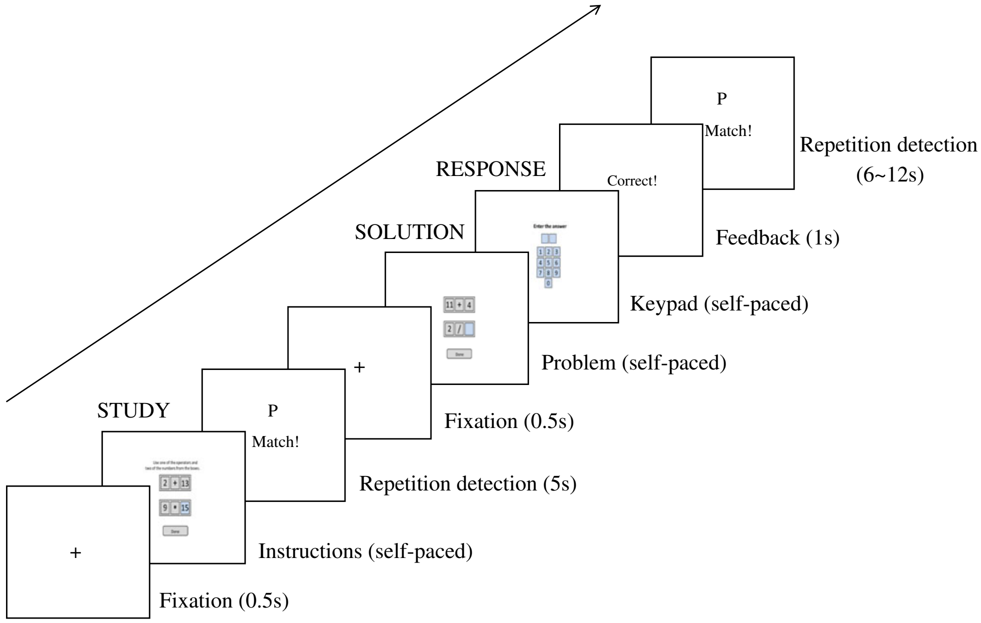

Fig. 2. Illustration of events for each trial, showing the three critical periods: study, solution, and response period.

during structural acquisition in the scanner. The remaining three sets of problems were the focus for analyses reported here. Each set of problems was performed during a functional acquisition in the scanner, with a period of rest between each functional scanner run/problem set. Participants interacted with problems by means of a mouse.

Before the experiment began, participants were given a brief test to make sure that they understood various terms that would appear during the study (e.g., what is the inverse operator of + ?, what are the diagonal two numbers on the given figure?). They were also given a few practice trials to become familiarized with the trial sequence and system interface. These practice problems were represented exactly like actual task items, but simply required copying numbers without true problem solving. At the beginning of the study, participants were instructed that their task is to figure out a problem-solving rule (what numbers to use and what operator to use for computation) on the instruction page and then apply that rule to solve a subsequent problem. They were also informed that the problem-solving rule would change for every trial. Participants were asked to respond as accurately and quickly as possible. The entire study took about 1 hr.

This study employed an event-related design with the goal of examining differences between example-instruction trials (Example) and verbal-instruction trials (Verbal). Figure 2 shows an illustration of events for each trial. The flow of a trial can be characterized as consisting of three separate periods: (a) a study period, in which the instructions (either example or verbal) were presented on the screen; (b) a solution period, in which a problem was presented and participants had to mentally solve by applying the instructed rule; (c) a response period, in which participants entered an answer using an on-screen numerical keypad. Both study period and solution period were preceded by a 0.5 s fixation and were self-paced. Participants were instructed to press a done button as soon as they had studied the instructions or computed the result. Thus, each trial in the scanner had the following sequence:

1. 0.5 s of fixation.
2. A self-paced study period: An instruction page was presented and participants had up to 15 s to study the instruction. Participants clicked a done button when they had finished encoding.
3. 5 s of repetition-detection task. In the repetition-detection task, letters appeared on the screen at a rate of 1/1.25 s. Participants were told to click a match button whenever the same letter appeared twice consecutively. This was intended to discourage participants from extending their encoding of the instructions and enable us to separate estimate of study and solution periods.
4. 0.5-s of fixation.
5. A self-paced solution period: A problem page was presented and participants had up to 15 s to solve the problem. Participants clicked a done button when they had finished their mental computation.
6. A self-paced response period: A response page was presented and participants had up to 5 s to enter an answer. A numerical keypad appeared on the screen and the system automatically accepted the participant's response as soon as two numbers were entered.

7. 1 s of feedback. A feedback page showed one of the messages, *correct*, *incorrect*, or *time's up* on the middle of the screen. There was no information provided other than correctness.
8. Repetition detection task whose duration varied randomly between 6 and 12 s. This task served to distract the participants from the main task and return brain activity to a relatively constant level. The variable delays reduced the collinearity between the periods and allowed for a better estimation of the corresponding brain activity.

In addition, each functional scanning run began with a fixation period and a repetition detection period.

Functional images were acquired using gradient-echo echo planar image (EPI) acquisition on a Siemens 3 T Verio Scanner using a 32-channel RF head coil, with 2 s repetition time (TR), 30 ms echo time (TE), 79° flip angle, and 20 cm field of view (FOV). The experiment acquired 34 axial slices on each TR using a 3.2 mm-thick, 64×64 matrix. This produces voxels that are 3.2 mm high and $3.125 \times 3.125\ \text{mm}^2$. The anterior commissure-posterior commissure (AC-PC) line was on the 11th slice from the bottom scan slice.

fMRI Analysis

Acquired images were preprocessed and analyzed using AFNI (Cox, 1996; Cox & Hyde, 1997). Functional images were motion-corrected using 6-parameter 3D registration. All images were then slice-time centered at 1 s and co-registered to a common reference structural MRI by means of a 12-parameter 3D registration and smoothed with an 6 mm full-width-half-maximum 3D Gaussian filter to accommodate individual differences in anatomy.

Our primary goal was to examine the effects of task (study vs. solution) and type of instruction (verbal vs. example) on brain activity. Imaging data were analyzed using a general linear model (GLM). For each participant, a regression model was constructed (first-level design matrix) consisting of seven model variables and a baseline model of an order-4 polynomial to account for general signal drift. Six of the model variables corresponded to the 2 (Example vs. Verbal instructions) by 3 (Study, Solution, and Response periods) in a trial. There was a single additional variable corresponding to the feedback period that concluded each trial, collapsed over problem types. The seven regressors were constructed by convolving the boxcar functions of these variables (on and off periods) with the standard SPM hemodynamic response function (Friston, Ashburner, Kiebel, Nichols, & Perry, 2011). Thus each regressor is a predictor reflecting the ideal BOLD response to varying durations of neural activity as expressed via the boxcar functions. Each regression yielded seven beta weights per voxel for each participant. Analyses focused on the study and solution periods. Group level analyses were performed on these first-level beta estimates. Both whole-brain exploratory analyses and predefined region of interest (ROI) analyses of average beta weight per region were conducted.

RESULTS

Behavioral Results

We first examined whether participants' accuracy differed between the example and verbal direction conditions. Overall, participants' accuracy was quite high (overall $M = 95.14\%$, $SD = 4.26$). To see the effect of instructional condition across three sets of problems, 2 (example vs. verbal direction) × 3 (set 1 vs. 2 vs. 3) repeated measures analysis of variance (ANOVA) was performed. There was a significant main effect of instructional condition, $F(1, 19) = 13.96$, $p = .001$, $\eta_p^2 = .424$. Participants solved more problems correctly when they were given a verbal direction ($M = 96.8\%$ $SD = 3.9$) than an example ($M = 93.5\%$, $SD = 5.4$). However, there was neither a main effect of problem set, $F < 1$, nor an interaction effect, $F < 1$. While significant, the overall effect of instructional condition was small. Subsequent analyses will focus on correct trials only.

Each trial consists of three periods: a study period, a solution period, and a response period. Accordingly, there were three types of latency data depending on the period. Figure 3 shows mean times for correct Example and Verbal trials across the three problem sets broken down by period within trial. A series of 2×3 ANOVAs were performed to see how latency varied by instructional condition over the three problem sets. During the instruction study period (Figure 3a), there was a significant mean difference between the example and verbal direction conditions, $F(1, 19) = 45.65$, $p < .0001$, $\eta_p^2 = .706$. Participants took longer studying examples ($M = 6.67$s, $SD = 1.85$) than studying verbal directions ($M = 4.90$s, $SD = 1.83$). Also, there was a significant main effect of problem set, $F(1, 19) = 17.58$, $p < .0001$, $\eta_p^2 = .481$, showing a trend that participants became faster as they had more practice. However, there was no instruction-by-set interaction, $F(1, 19) = 1.02$, $p = .325$, $\eta_p^2 = .051$. Different patterns of results were observed in the solution (Figure 3b) and response (Figure 3c) periods. In both periods, there was not a significant main effect of instructional condition, $F\text{s} < 1$, but a significant main effect of problem-set. Participants took longer in initial problem sets than later problem sets in both the solution period $F(1, 19) = 14.10$, $p = .001$, $\eta_p^2 = .426$, and the response period, $F(1, 19) = 28.28$, $p < .0001$, $\eta_p^2 = .598$. Also, neither solution nor response period showed interaction effects, $F\text{s} < 1$.

Imaging Results—Predefined Regions

Results of the analyses for the four predefined regions are shown in Table 2. We focused on only the left side of

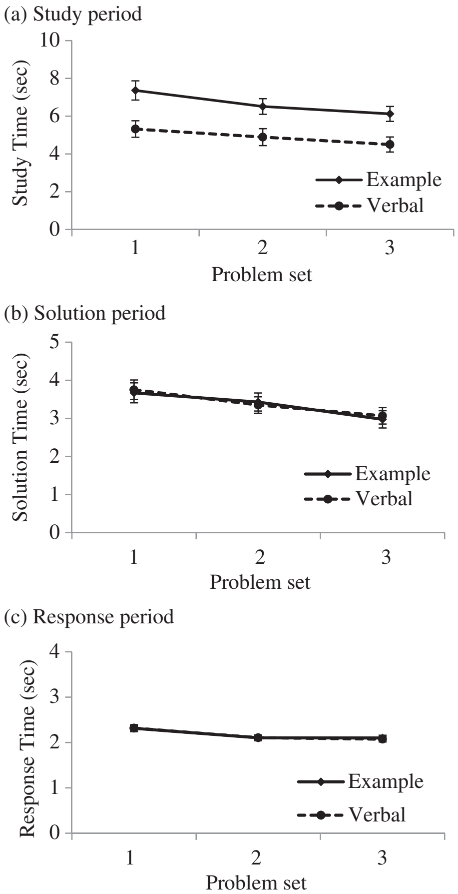

Fig. 3. Mean time for correctly solved problems between example and verbal trials across three sets of problems. (a) study period; (b) solution period; (c) response period. Error bars represent 1 SE of mean.

the brain, where we have found stronger effects in past research. Their patterns of activation are shown in Figure 4. For each of the four predefined regions, we performed a series of 2 × 2 within-subjects ANOVAs to examine the effects of instructional condition (example vs. verbal direction) and period (instruction study vs. solution). Table 2 shows main effects and interaction effects found in these four predefined regions. There were significant effects of instructional condition: the example condition (vs. verbal direction) showed significantly higher engagement in the HIPS (example $M = 1.17\%$, verbal $M = 0.96\%$) and the LIPFC (example $M = 1.08\%$, verbal $M = 0.90\%$) regions. None of the four predefined regions showed higher engagement in the verbal direction condition.

Regarding the effect of period, there was greater activation for study period (vs. solution period) in the AG (study $M = 0.44\%$, solution $M = 0.18\%$) and the Fusiform (study $M = 0.78\%$, solution $M = 0.45\%$) regions. The AG region showed a significantly greater activity in the study period than in the solution period in both the example condition,

Table 2

2 × 2 ANOVA Results With Beta Weights of Correct Trials (F's with dfs of 1 and 19)

ROI	*Instruction [Example > Verbal]*	*Period [Study > Solution]*	*Interaction*
HIPS	15.12***	(11.23)**	1.89
LIPFC	20.72***	(13.62)**	13.66**
AG	0.06	9.66**	0.24
Fusiform	2.67	18.13***	0.03

Note: Values in parentheses indicate an opposite direction of the stated main effect.
*$p < .05$; **$p < .01$; ***$p < .001$.

$t(19) = 3.45$, $p = .003$, and the verbal direction condition, $t(19) = 2.13$, $p = .046$. Likewise, the Fusiform region showed a significantly greater activity during the study period in both example condition, $t(19) = 3.87$, $p = .001$, and verbal direction condition, $t(19) = 3.25$, $p = .004$. The other two predefined regions showed significant effects in the opposite direction: HIPS (study $M = 0.97\%$, solution $M = 1.17\%$) and LIPFC (study $M = 0.84\%$, solution $M = 1.13\%$). The HIPS region showed a significantly greater activity in the solution period than study period in both the example condition, $t(19) = 2.41$, $p = .026$, and the verbal direction condition, $t(19) = 2.83$, $p = .011$. Similarly, the LIPFC region showed a significantly greater activity in the solution period than study period in the verbal direction condition, $t(19) = 4.40$, $p < .001$, but such difference was not statistically significant in the example condition, $t(19) = 1.96$, $p = .065$, reflecting the interaction between the instruction and the period.

The interaction between instruction and period was found only in the LIPFC region where activation was lower in the study period for verbal directions than the other three conditions. In the study period, brain activity was significantly greater in the example condition than the verbal direction condition, $t(19) = 5.91$, $p < .001$. However, in the solution period, such significant differences disappeared between the two instructional conditions, $t(19) = 0.76$, $p = .457$.

As predicted, participants showed greater activation during the instruction study period in the Fusiform, reflecting the need for detailed visual examination, but greater activity during problem solving in the LIPFC and HIPS reflecting the need for retrieval of arithmetic facts and numerical processing. Again as predicted, the LIPFC did show an interaction, where its activity was least during the study of verbal instructions. The HIPS also showed an effect in this predicted direction such that a significantly greater activity was found in the example condition than the verbal direction condition during the study period, $t(19) = 5.02$, $p < .001$, but this difference disappeared during the solution period, $t(19) = 1.62$, $p = .121$. However, an interaction effect was not statistically significant. Also as predicted, none of these regions showed

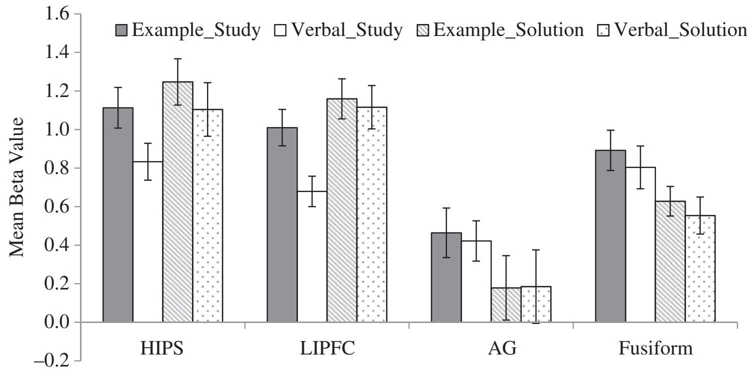

Fig. 4. Mean beta values of the four predefined regions in study and solution period between example and verbal direction conditions. Error bars represent 1 SE of mean.

a significant difference between verbal and example instruction during the solution period ($t(19) = 1.62$, $p = .121$ for HIPS, $t(19) = 0.76$, $p = .457$ for LIPFC) whereas the HIPS and LIPFC did during the study period ($t(19) = 5.02$, $p < .001$ for HIPS and $t(19) = 5.91$, $p < .001$ for LIPFC).

Imaging Results—Exploratory Regions

For completeness, exploratory analyses were performed looking for significant effects of instructions during the two periods in order to determine what other brain areas, if any, might show effects. These analyses looked for regions of at least 21 contiguous voxels that showed a voxel-wise significance of 0.0005 for the difference between the contrast of the described variables. Using these values results in a brain-wise significance estimated to be less than 0.01 by simulation (Cox, 1996; Cox & Hyde, 1997).

No region showed a significant effect of instructional condition during the solution period, but 11 regions emerged for the study period (see Table 3). Figure 5 shows the activity of those regions in the study and solution period between example and verbal direction conditions. Among these regions, six regions showed greater activity while studying examples whereas five regions showed greater activity while studying verbal directions. Figure 6 displays the regions, showing a significant effect of instruction type during the study period. In this figure, the blue regions show greater activity while studying examples whereas the red regions show greater activity while studying verbal directions. Consistent with our predefined region analysis, the region (ROI 1) that overlapped with our predefined LIPFC region showed greater activity in the example condition. Further, even though the effect in the predefined HIPS did not produce a significant interaction, there was an overlapping parietal region (ROI 3) that did show the predicted significant interaction.

Although none of our predefined regions showed a significantly greater activity in the verbal direction than example condition, four regions showed greater activity while studying verbal directions. These regions included motor (ROI 8 and 9 in Table 3) and visual (ROI 10) regions.

GENERAL DISCUSSION

This article investigated the effect of two different forms of instruction on learning in mathematical problem solving and identified how these two methods affected the activity of relevant brain regions. In particular, we contrasted learning from examples with learning from verbal directions. Participants studied either an example or a verbal direction and then applied what they learned to a subsequent problem. Behavioral data showed that participants solved problems slightly more accurately when they were given verbal directions than examples. Also, they spent less time studying verbal directions compared to examples. In the introduction, however, we reviewed behavioral research showing success with both examples and verbal directions. It seems reasonable to conclude that whether one instructional modality is better than another depends on the exact nature of the material and the details of the instruction. One modality does not appear to have an inherent advantage over the other. The advantage of verbal directions was small in our experiment and our real interest was in how these two types of instruction are processed.

The imaging results showed distinctive neural signatures between the two instructional conditions. Studying an example (vs. verbal direction) tended to increase activity in the prefrontal regions including the LIPFC and parietal regions including the HIPS. These regions are known to be engaged in mathematical problem solving. While studying an example, participants have to discover a rule by combining presented numbers and operators so that the result value corresponds to the value of the answer box. This process necessarily requires some amount of mathematical

Table 3
Regions Identified From Exploratory Analysis Using the Verbal Direction Versus Example Contrast During the Study Period

Regions		*Voxel count*	*Brodmann areas*	*Coordinates (x, y, z)*	*Instruction (Example > Verbal)*	*Period (Study > Solution)*	*Interaction*
(a) Example > Verbal direction							
1	L Middle Frontal Gyrus, Superior Frontal Gyrus	111	6	−24, 2, 54	48.22***	(27.81)***	4.79*
2	R Middle Frontal Gyrus	26	6	26, −1, 52	15.69***	(22.32)***	7.97*
3	L Inferior/Superior Parietal Lobule, Precueneus	381	40,7	−36, −52, 44	27.43***	(18.47)***	4.49*
4	M Medial Frontal Gyrus, Cingulate Gyrus	59	32,6,8	−1, 19, 43	18.78***	(37.90)***	14.36**
5	R Inferior Parietal Lobule, Precuneus	55	40	34, −51, 41	12.41**	(2.26)	11.14**
6	L Middle/Inferior Frontal Gyrus, Precentral Gyrus	261	9,46,10	−44, 22, 25	36.24***	(41.26)***	27.00***
(b) Verbal direction > Example							
7	M Medial Frontal Gyrus, Superior Frontal Gyrus	56	6	−3, −6, 58	(14.60)**	2.67	7.00*
8	L Precentral/Postecentral Gyrus	112	4,3	−43, −15, 48	(15.93)***	15.61***	16.07***
9	R Precentral/Postecentral Gyrus	53	4,3	34, −16, 40	(21.28)***	47.83***	2.98
10	L/M/R Cuneus, Lingual Gyrus, Posterior Cingulate, Middle Occipital Gyrus, Culmen, Parahippocampal Gyrus	1,604	18,19,30,17,31	1, −75, 9	(19.76)***	137.80***	51.41***
11	L Anterior Cingulate	22		−18, 31, 6	(0.13)	12.43**	36.79***

Note: The last three columns report main effects of instruction, period, and instruction by period interaction (Fs with dfs of 1 and 19). Values in parentheses indicate an opposite direction of the stated main effect.
**p* < .05. ***p* < .01. ****p* < .001.

computation and this appeared to increase activity in these regions. Increased activity in the LIPFC seemed to support such numerical processing that was required while studying an example. The importance of the LIPFC region has been emphasized in the models of equation solving (Anderson, 2005; Ravizza, Anderson, & Carter, 2008) and mental multiplication (Rosenberg-Lee et al., 2009) that were based on the ACT-R theory (Anderson, 2007; Anderson et al., 2004). Many other studies also have shown that the LIPFC is involved in mathematical cognition (e.g., Krueger et al., 2008; Qin et al., 2004; Ravizza et al., 2008; Sohn et al., 2004), and more specifically in retrieval of arithmetic facts in mathematical problem solving (Danker & Anderson, 2007; Menon et al., 2000). Increased activity in the HIPS also seemed to support numerical processing while participants were studying an example. This is consistent with numerous previous studies that have shown the role of HIPS in mathematical tasks and processing of quantity (Cohen-Kadosh et al., 2008; Dehaene, 1997; Dehaene & Cohen, 1997; Dehaene et al., 2003; Lee et al., 2014; Rosenberg-Lee et al., 2009).

Although none of our predefined regions showed greater activity while studying verbal directions than examples, we identified a few regions that showed increased activity while studying verbal directions (ROIs 7–10 in Table 3). Greater activity in those regions appeared to be because of processing written instructions. While reading verbal directions, participants had to process visual information and this perhaps increased activity in the visual region. Also, increased activity in the pre- and postcentral regions perhaps reflects motor activity associated with subvocalization of the rule.

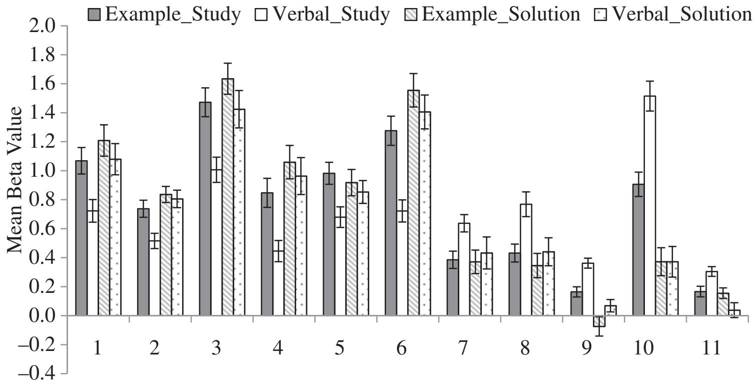

Fig. 5. Mean beta values of the exploratory regions in study and solution period between example and verbal direction conditions. Error bars represent 1 SE of mean. See Table 3 for identities of the number code on x-axis.

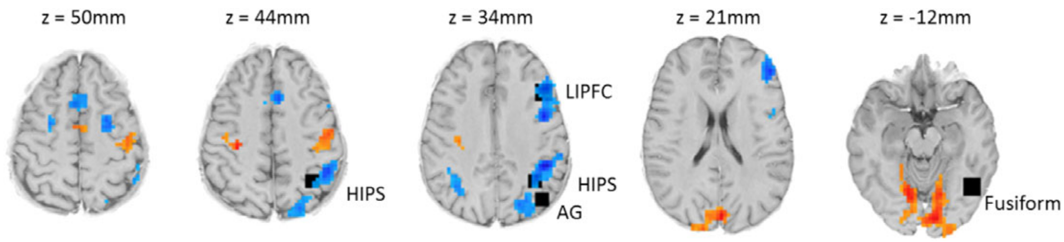

Fig. 6. Exploratory regions showing a significant example versus verbal direction contrast during the study period. The black squares show the predefined regions in the experiment. The blue regions show greater activity while studying examples whereas the red regions show greater activity while studying verbal directions. The z coordinates for a brain slice (radiological convention: image left = participant's right) is at $x = y = 0$ in Talairach coordinates.

The current research also identified distinctive neural signatures between the study and solution periods. The Fusiform and AG regions showed greater activity during the study period whereas the LIPFC and HIPS regions showed greater activity during the solution period. The greater activity during the study period in the Fusiform region appeared to be because reading the written instruction or studying the example involved more detailed visual processing than simply reading the numbers and operators required to produce the solution. Numerous studies have shown that the Fusiform area plays a critical role in visual processing of attended information (e.g., Grill-Spector, Knouf, & Kanwisher, 2004; Leff et al., 2001; McCandliss, Cohen, & Dehaene, 2003; Price & Mechelli, 2005; Shaywitz et al., 2002; Turkeltaub et al., 2003). Figuring out how to solve a problem given an example or verbal direction requires visual attention and careful examination of instructions, and thus it could have caused greater activity during the study period than the solution period.

In the introduction we noted that one might have predicted greater activity in the AG during the solution period given claims that it is involved in retrieval of arithmetic facts (Dehaene et al., 2003; Grabner et al., 2009; Schmidthorst & Brown, 2004) and verbal processing of numbers (Anderson et al., 2014; Dehaene et al., 2003). However, we also noted claims for its role in metacognition and comprehension (Anderson et al., 2011; Wintermute et al., 2012), and its greater activity during study suggests that this is its principal function in our study. While it does seem to be more engaged in processing the instruction, the lack of a difference between the example and verbal direction conditions suggests that its activity is not particularly tied to language processing. During the study period, participants had to figure out how to solve a problem, and this might have increased metacognitive activity and in turn greater activity in the AG region.

Most interestingly, both of our predefined LIPFC region and exploratory regions that overlapped with LIPFC and HIPS showed significant instruction by period interactions. Consistent with our original hypotheses, the LIPFC region showed reduced activity while studying verbal directions (i.e., study period of verbal direction condition) than the other three conditions (i.e., study period of example condition, solution period of example condition, and solution period of verbal direction condition). When studying verbal directions, participants had to focus on comprehending the written instructions and did not have to do any computations. In contrast, in all three other conditions participants needed to retrieve arithmetic facts to do arithmetic computations. This increased the activity in the LIPFC region in these three conditions whereas it was not the case for the study period of verbal direction condition.

As we intended for the purposes of this study, examples and verbal directions were nearly equally effective in enabling students to solve later problems. Putting both behavioral and fMRI results together, the most important finding is that neural activation appeared quite different between two different types of instruction during the study period, but such differences disappeared during the solution period. This suggests that different instructional methods may result in a different encoding process; however, when that encoding is transformed to an actual problem-solving behavior, people seem to show a similar execution process. Overall, the results suggest that instruction type did not matter after getting to an understanding. This is consistent with the idea that what is learned is more important than how it is learned (Klahr & Nigam, 2004; Lee et al., 2014).

These results do have some implications about when example-based instruction may be more appropriate. A common distinction in discussing educational goals is between imparting procedural and conceptual knowledge (Bisanz & LeFevre, 1992; Hiebert & LeFevre, 1986; Rittle-Johnson & Alibali, 1999; Rittle-Johnson, Siegler, & Alibali, 2001). Procedural knowledge is typically defined in terms of the ability to solve specific problems. On the other hand, conceptual knowledge is considered to involve understanding relationships among elements of the domain abstracted from their connection to any problem-solving procedure. One might have thought that students would just process the abstract structure of the relationships in the examples, but our results suggest that they engage in a process very much like the process they engage in while solving the problem. Thus, this research is consistent with the proposal that processing an example focuses more on conveying procedural knowledge than processing a verbal direction. One might be tempted to conclude the opposite is true for processing of verbal instruction—that it is better than example-based instruction for conceptual knowledge. However, the current study does not provide evidence on this because it did not involve a conceptual test.

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