2

Analysis of Student Performance with the LISP Tutor

John R. Anderson
Carnegie-Mellon University

The goal of this chapter is to present our first detailed analysis of student performance with the LISP tutor. First, we describe a little of our general theoretical orientation to the issues of intelligent tutoring. Second, we provide a description of the essential features of the operation of the LISP tutor. Third, we give some general description of characteristics of the data that are obtained with the LISP tutor.

INTELLIGENT TUTORING AND ITS RELATION TO COGNITIVE THEORY

Research on intelligent tutoring serves two goals. The obvious goal is to develop systems for automating education. Private human tutors are very effective (Bloom, 1984), and it would be nice to be able to deliver this effectiveness without incurring the high cost of human tutors. However, a second and equally important goal is to explore epistemological issues concerning the nature of the knowledge that is being tutored and how that knowledge can be learned. We take it as an axiom that a tutor is effective to the extent that it embodies correct decisions on these epistemological issues.

We chose intelligent tutoring as a domain for testing out the ACT* theory of cognition (Anderson, 1983). It was a theory that made claims about the organization and acquisition of complex cognitive skills. The only way to adequately test the sufficiency of the theory was to interface it with the acquisition of realistically complex skills by large populations of students. When we read *Intelligent Tutoring*, edited by Sleeman and Brown (1982), it became apparent that the book's authors were explicitly or implicitly performing such tests of theories of cognition and that it was an appropriate methodology for testing the ACT* theory. Fundamentally, the tutoring methodology is predicated on the assumption that one understands a skill and its acquisition. The success of the tutor constitutes a direct test of the sufficiency of the underlying theory.

The ACT* theory has been used to construct performance models of how students actually execute the skills that are to be tutored and learning models of how these skills are acquired. The performance model is used in a paradigm we call *model tracing* in which we try to follow in real time the cognitive states the student goes through in solving a problem. The power of our tutoring approach depends critically on the success of our model-tracing apparatus to correctly interpret the cognitive states of students. When we interrupt students to provide instruction, that instruction is given with respect to an assumed mental state. If this model's assumptions are wrong, the instruction will be off the mark.

The LISP tutor (Anderson & Reiser, 1985) was developed as an instantiation of this model-tracing methodology and serves to test our theory of skill acquisition (Anderson, 1982; Anderson, Farrell, & Sauers, 1984) in two ways. First, it is a sufficiency test of the theory. The fact that a system of this variety does serve to teach LISP programming skills stands as a general confirmation of the theory. Second, it is also a tool to test predictions of the theory.

Although our research is in LISP programming and its tutoring, we are using this as a vehicle to test some fundamental issues about the nature of problem-solving skills and its acquisition. Among these issues are the following:

1. *Skill Representation*. How should a skill be presented? ACT* assumes a representation as a set of production rules.
2. *Procedural Versus Declarative Knowledge*. What is the relationship between the declarative knowledge (which is the original instruction) and the highly proceduralized form that it finally achieves?
3. *Performance Limitations*. How do fundamental performance limitations like working-memory limitations impact on skill performance and skill acquisition?
4. *Organization and Control*. How is the knowledge underlying problem-solving skill organized and controlled to permit coherent problem solving?
5. *Skill Modification*. How is one's knowledge modified to effectively reflect experience? This issue is closely tied up with the issue of feedback.
6. *Mechanisms of Skill Acquisition*. Last but hardly least, what are the fundamental mechanisms of skill acquisition?

LISP programming is an excellent domain for studying these issues because it offers a complex but relatively well-understood domain. The tutor is an excellent tool because it brings control and experimental rigor to what would otherwise be a rather free-form learning experience.

THE LISP TUTOR

The LISP tutor currently teaches a full-semester, self-paced course at Carnegie-Mellon University. It covers all the basic concepts in LISP. It is the first instance of a practical piece of intelligent tutoring being widely used, and it has been shown to lead to improvement in performance. Roughly, students working on problems with the LISP tutor get one letter grade higher on final exams of general competence than students not working with the LISP tutor (Anderson & Reiser, 1985). It should also be noted that students working with private human tutors have been shown to outperform students with the LISP tutor. So it is by no means a utopian system, but it can claim some pedagogical effectiveness.

Table 2.1 contains a dialogue with a student coding a recursive function to calculate factorial. This does not present the tutor as it really appears. Instead, it shows a "teletype" version of the tutor where the interaction is linearized. In the actual tutor the interaction involves updates to various windows. In the teletype version, the tutor's output is given in normal type whereas the student's input is shown in bold characters. These listings present "snapshots" of the interactions; each time the student produces a response, his input is listed along with the tutor's response (numbered for convenience). The total code as it appears on the screen is shown, although the student has added only what is different from the previous code (shown in boldface type). For instance, in Line 2 he has added "zerop" as an extension of "(defun fact (n) (cond ((. . . . "

In the first line, when the subject typed "(defun", the template (defun <name> <parameters> <body>) appeared. The terms in <—> angle brackets denote pieces of code he will supply. The subject then filled in the <name> slot and the <parameters> slot and had started to fill in the <body> slot. Note that at all points parentheses are balanced and syntax is checked.

Although the student has some difficulty with the syntax of the conditional tests in Lines 1 and 2, he basically codes the terminating case for the factorial function correctly. Typically, we find students have little difficulty with terminating cases but have great difficulty with recursive cases. The dialogue after Line 3 illustrates how the tutor guides the student through a design of the recursive function. Basically, it leads the student to construct a couple of examples of the relationship between fact (*n*) and fact (*n–1*) and then gets the student to identify the general relationship. Figure 2.1 shows the screen image at a critical point in the design of this function.

The dialogue after this point shows two errors students make in defining recursive functions. The first, in Line 4, is to call the function directly without

TABLE 2.1

Define the function fact. It takes a number argument, n and computes the factorial of n, which is the result of multiplying n × n−1 × n−2 . . . × 1. For instance, (fact 3) = 3 × 2 × 1 = 6. By definition, the factorial of 0 is 1.

```
1. (defun fact (n)
     (cond ((equal) <ACTION>)
       <RECURSIVE-CASE>))
```

I assume that you are trying to test if the number argument is equal to the number zero. There is a special function for testing equivalence to zero. Try to use that function instead of EQUAL.

What code can you write to test if the condition for the terminating case is true? Try writing a predicate.

```
2. (defun fact (n)
     (cond ((zero <ACTION>)
       <RECURSIVE-CASE>))
```

Did you mean zerop (y or n)? **y**

```
3. (defun fact (n)
     (cond ((zerop n) 1)
       <RECURSIVE-CASE>))
```

Is the value of fact in the recursive cases calculated by:

PRESS:	IF YOU WANT TO:
1	Calling fact recursively with n.
2	Multiplying n by fact of n minus 1.
3	Getting 1 2 . . . n and then multiplying the results.
4	have the tutor choose

Menu choice: **3**

That is a reasonable way to think of doing factorials but it is not a plan for a recursive function. Since you seem to be having trouble with the recursive cases let us work through some examples and figure out the conditions and actions for each of these cases.

The recursive relationship is the relationship between fact called with n and fact called with:

PRESS:	IF YOU WANT TO:
1	n minus 1
2	n equal to 0
3	n plus 1
4	have the tutor choose

Menu choice: **1**

We will consider some concrete examples. We will characterize the recursive relationship between fact called with 1 and fact called with 0

What is the value of (fact 1)?
Answer: **1**
What is the value of (fact 0)?
Answer: **1**

EXAMPLES

	fact (n)	fact(n − 1)
A.	(fact 1) = 1	(fact 0) = 1

We will consider some more concrete examples. We will characterize the recursive relationship between fact called with 3 and fact called with 2

What is the value of (fact 3)?
Answer: **6**

What is the value of (fact 2)?
Answer: **2**

EXAMPLES

	fact (n)	fact (n − 1)
A.	(fact 1) = 1	(fact 0) = 1
B.	(fact 3) = 6	(fact 2) = 2

Since there is only one recursive case characterized by the examples you have just worked out the condition in this case can be just coded at T. We will just focus on characterizing the action in this case.

In examples A and B what do you have to do to get the result of fact called with n?

PRESS	IF YOU WANT TO:
1	Multiply n by one less than n.
2	Multiply n by fact of one less than n
3	Add n to the result of fact called with one less than n.
4	have the tutor choose

Menu choice: **2**

We have now worked through a series of examples that illustrate the recursive relation. We will now return to the goal of trying to produce LISP code that specifies the recursive relation.

```
4. (defun fact (n)
     (cond ((zerop n) 1)
       (t (fact))))
```

You will have to use fact at some point but right now you want to multiply.

```
5. (defun fact (n)
     (cond ((zerop n) 1)
       (t (times n (fact n)))))
```

You cannot just code n at this point. What you want to do is subtract one from n.

```
6. (defun fact (n)
     (cond ((zerop n) 1)
       (t (times n (fact (difference)))))
```

Difference would work in this case but it is more appropriate to use the function SUB1 when you are subtracting 1 from a number.

```
7. (defun fact (n)
     (cond ((zerop n) 1)
       (t (times n (fact (sub1 n))))))
```

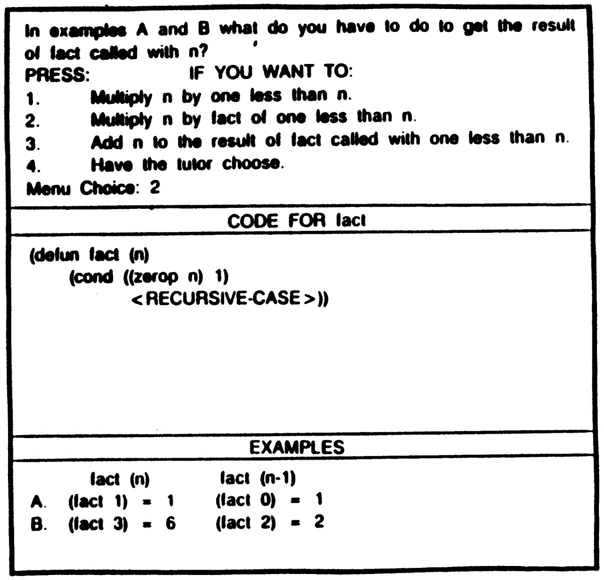
—YOU ARE DONE. TYPE NEXT TO GO ON AFTER—
—TESTING THE FUNCTIONS YOU HAVE DEFINED—

(continued)

TABLE 2.1 *(continued)*

THE LISP WINDOW

```
= > (trace fact)
(fact)
= > (fact 3)
1 <Enter> fact (3)
|2 <Enter> fact (2)
| 3 <Enter> fact (1)
| |4 <Enter> fact (0)
| |4 <Exit> fact 1
| 3 <Exit> fact 1
|2 <Exit> fact 2
1 <Exit> fact 6
6
```



In examples A and B what do you have to do to get the result of fact called with n?

```
PRESS:              IF YOU WANT TO:
1.        Multiply n by one less than n.
2.        Multiply n by fact of one less than n.
3.        Add n to the result of fact called with one less than n.
4.        Have the tutor choose.
Menu Choice: 2
```

CODE FOR fact

```
(defun fact (n)
    (cond ((zerop n) 1)
          <RECURSIVE-CASE>))
```

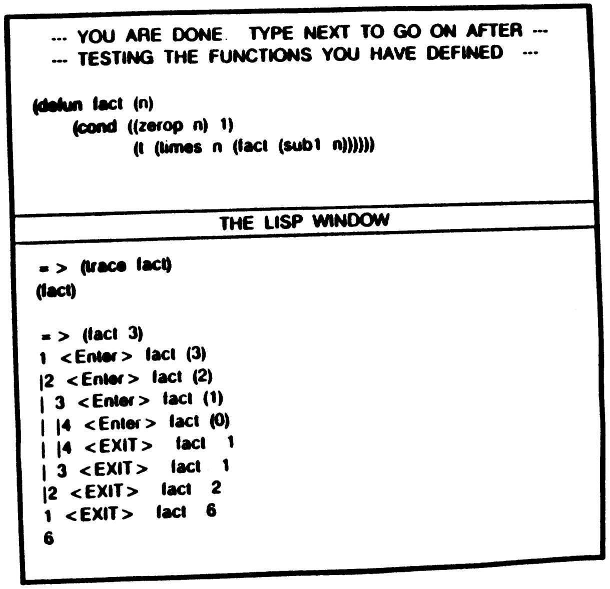
EXAMPLES

```
      fact (n)         fact (n-1)
A.  (fact 1) = 1     (fact 0) = 1
B.  (fact 3) = 6     (fact 2) = 2
```

FIG. 2.1. A representation of the screen image after line 3 in Table 2.1

combining the recursive call with other elements. The second, in Line 5, is to call the function recursively with the same argument rather than a simpler one.

After the student finishes coding the function he goes to the LISP window and experiments. He is required to trace the function, and the recursive calls embed and then unravel. Figure 2.2 shows the screen image at this point with the code on top and the trace below it.



```
--- YOU ARE DONE.  TYPE NEXT TO GO ON AFTER ---
--- TESTING THE FUNCTIONS YOU HAVE DEFINED  ---

(defun fact (n)
    (cond ((zerop n) 1)
          (t (times n (fact (sub1 n))))))
```

THE LISP WINDOW

```
= > (trace fact)
(fact)

= > (fact 3)
1  <Enter>  fact  (3)
|2  <Enter>  fact  (2)
| 3  <Enter>  fact  (1)
| |4  <Enter>  fact  (0)
| |4  <EXIT>   fact   1
| 3  <EXIT>   fact   1
|2  <EXIT>   fact   2
1  <EXIT>   fact   6
6
```

FIG. 2.2. The final screen image at the end of the dialogue in Table 2.1

Features of the Model-Tracing Methodology

The example just shown illustrates a number of features of the model-tracing methodology:

1. The tutor constantly monitors the student's problem solving and provide direction whenever the student wanders off a solution path.
2. The tutor tries to provide help with both the overt parts of the proble solution and the planning. However, to address the planning, a mechanism had to be introduced in the interface (in this case menus) to allow th student to communicate the steps of planning.
3. The interface handles details like syntax checking, which are irrelevant t the problem-solving skill being tutored.
4. The interface is highly reactive in that it does make some response t every symbol the student enters.

The Mechanics of Model Tracing

Sitting within the tutor is a production system consisting of hundreds of ideal an buggy rules. The following are examples of a production rule that codes APPEN and two bugs. Associated with each bug is an example of the feedback we woul present to the student should the student display that bug:

Production Rule in Ideal Model:

```
        IF    the goal is to merge LIST1 and LIST2
                into a single list
      THEN    use the function APPEND and set
                subgoals to code LIST1 and LIST2
```

Related Bugs:

```
        IF    the goal is to merge LIST1 and LIST2
                into a single list
      THEN    use the function LIST and set
                subgoals to code LIST1 and LIST2
```

You should combine the first list and the second list, but LIST is not the right function. If you LIST together (a b c) and (x y z), for example, you will get ((a b c) (x y z)) instead of (a b c x y z). LIST just wraps parens around its arguments.

```
        IF    the goal is to merge LIST1 and LIST2
                into a single list
                and LIST1 = LIST2
      THEN    use the function TIMES and set
                subgoals to code LIST1
                and the number 2
```

You want to put together two copies of the same list, but you can't make two copies of a list by using the function TIMES. TIMES only works on numbers. You should use a function that combines two lists together.

Altogether we have over 1200 productions (correct and buggy) to model student performance in our lessons, which cover all the basic syntax of LISP, design of iteration and recursive functions, use of data structures, and means–ends planning of code.

THEORETICAL PREMISES UNDERLYING THE LISP TUTOR

The LISP tutor is predicated on a number of assumptions about the cognitive architecture, about the nature of a complex skill like programming, and the nature of its acquisition. The actual student models are implemented in the GRAPES production system, which is a partial simulation of the ACT* theory, and the tutoring interactions are based on assumptions about what the critical factors are underlying skill performance and execution. Without claiming to be exhaustive, the following are among the critical assumptions underlying execution of the LISP tutor:

1. *Production Rule Decomposition*. A skill like programming can be decomposed into an independent set of production rules.

2. *Skill Complexity*. The number of rules underlying such a skill is large—at least in the hundreds and perhaps in the thousands. This is the perspective of the knowledge engineering tradition in AI and implies detailed task analysis is a prerequisite to skill acquisition. It also implies that knowledge is highly specific.

3. *Hierarchical Goal Organization*. All the productions are organized by a hierarchical goal structure, which is traversed in a top–down left-to-right discipline.

4. *Declarative Origins of Knowledge*. All knowledge begins in some declarative representation, typically acquired from instruction or example. Thus, in the LISP tutor we always precede practice with a terse exposition designed to provide the critical declarative knowledge.

5. *Compilation of Procedural Knowledge*. Use of declarative knowledge is inefficient. There is a knowledge compilation process that converts the declarative knowledge into an efficient procedural form specific to a particular use. Knowledge compilation requires the knowledge to be actually used in the content of problem solving. It is by examining the trace of the problem solution that the knowledge-compilation process decides how to cast the production that it produces.

6. *Conscious Correction of Knowledge*. Errors in knowledge do not become automatically corrected with experience. They require that the subject form a declarative representation of the mistake and act to correct the mistake. Thus, the emphasis in the tutor is on explaining the difference between correct and incorrect options.

7. *Centrality of Working Memory Limitations*. A production system operates by matching information in its working memory. In the ACT* theory this working memory is the active portion of long-term memory. This is basically the students' immediate goal, what is attended to in the environment and their strong associates. Activation has a rapid decay; so unattended information quickly drops out of memory. The major performance factor (in contrast to lack of knowledge factor) limiting learning in ACT* is the failure to keep active in working memory all the information for a mental compilation to apply correctly. This view about the major limitation on LISP performance is supported in the research of Anderson and Jeffries (1985). They found that novice errors in LISP were largely slips and not the result of lack of knowledge. Moreover, these slips increased with working-memory load. Working-memory limitations impact on learning as well as performance. Because productions are compiled from declarative representations,

working-memory failures slow down learning and cause incorrect things to be learned.

8. *Minor Relations Between Strength and Learning*. As declarative knowledge or procedural knowledge is practiced it is strengthened. Stronger declarative knowledge is the more active and hence is more likely to be in working memory. Stronger productions are the more rapidly matched to what is in working memory. This means that it is more likely that well-encoded knowledge would overcome the limitations of working memory and would successfully apply. The first-order effect of strength would be on speed of performance, but it would have a second-order effect on working memory (and basically, accuracy). Although these strength factors do affect speed and accuracy performance measures, they should have little direct effect on production learning. Even when strength affects maintenance of information in working memory, these strength effects would occur after productions are learned (at least in the LISP tutor).

In total, these eight assumption constitute some profound claims about the course of skill acquisition. They also paint a rather simple picture of the process. The critical question concerns what the actual data have to say about the theory.

DATA ANALYSIS

As students interact with the LISP tutor we collect a total record of all of their responses and the time at which they complete these responses. A student's response is defined as something the tutor reacts to. Usually this amounts to a LISP symbol, when typing code, or a menu selection. We do not collect data at the level of inter-keystroke times; that level of data collection would be just too voluminous. We also record the times at which the tutor prints prompts and the identities of these prompts. Finally, in the data files we have records of the correct or buggy productions that the LISP tutor ascribed to students responses.

These data can easily be transformed into a form wherein we organize the data by the sequence of productions that the tutor assumed fired and associate times with the firing of each production. This amounts to ascribing a theoretical interpretation to the data in terms of the simulation program used by the tutor. This is a level of analysis that graduate students used to spend dissertations to achieve for a few subjects solving a few problems. We can achieve it automatically for a class full of students doing a semester's worth of work.

A key feature of the LISP tutor is that it keeps students on a correct path of problem solution. The tutor may be prepared to follow the student on many hundreds of ways of solving a problem, but this is much less than the thousands of ways, mostly incorrect, that students have been observed trying to solve a problem. At any point in time there is a set of possible next correct productions that the tutor is prepared to have the student execute. One of a possible set of things can happen:

1. The student generates an action that matches the action produced by one of the correct productions. The tutor assumes the production that generated this action is the one that fired in the student's head and continues to monitor for the production that follows that.
2. The student makes an error, the tutor responds to that error with feedback, and then the student generates an action that corresponds to a correct production. The tutor assumes that the feedback enabled the student to figure out the correct answer, and the student is back on track.
3. The student asks for the next step either immediately or after an error. The tutor provides the student with an explanation of the correct step and then provides the piece of code that corresponds to that step. The assumption is again that this explanation was sufficient to get the student back on track and the student is in the same mental state as the tutor.
4. The student generates three errors. In this case the tutor offers the same explanation as it would have had the student requested it and provides the next correct action.

The major complication hidden in this description concerns a dichotomy in the types of errors emitted. About 80% of the errors match buggy productions in the LISP tutor, and it is able to generate feedback specific to that error. The other 20% of productions are not matched, and the feedback is a default ("I don't understand that"). The majority of the undiagnosed responses are clearly erroneous on the student's part. Only on rare occasions do we observe a student with a solution that the tutor has not thought of.

The underlying assumption in these interactions is that before doing the next piece of the problem the student and the tutor are in the same mental state. From informal observations we know there are occasions when this is not true, for instance, when the student either misunderstands the problem statement or the feedback given by the tutor. This means that there is a certain noise built into our error attribution. We attribute an error to production applying in state X whereas a different production might be applying in state Y. It is difficult to know the magnitude of this "noise" in the data and how it compares with noise in other data. One test is the reliability and interpretability of the data obtained with the LISP tutor. Any experiment has noise in the data, and as in any experiment, we use the ratio of variance between conditions to variance within conditions to decide what effects could not be due to experimental noise.

Given this data base, there are two basic categories of data to collect from the LISP tutor: error measures and time measures. Both of these categories break down into two basic subtypes. For errors, we can calculate the probability of making an error on a production or the total number of errors made on a production. In calculating total number of errors, we score requests for the answer as the maximum, three errors. With respect to time, we can calculate the time for the correct answer to appear irrespective of the number of intermediate errors and whether the answer was provided by the student or the system. This provides a kind of speed through the problem measure. Alternatively, we can restrict our analysis to cases where the first answer is correct.

There are three categories of correct productions. There are productions that do not produce any overt action on the student's part. There are productions whose actions correspond to menu selections. And there are productions whose actions produce code. We restrict our analysis to the third kind and to those situations where the previous production was also a code-producing production. Measuring time from the end of the student's action of the previous production to the end of the action associated with current production gives us a fairly clean measure of time for the production to fire and time for the student to interact with the interface and enter the result. Even in this case we are looking at measures that include a lot of extraneous time such as typing. The actual times we are reporting are much larger than typically associated with production firing.

The data we are analyzing comes from 34 students who were taking LISP in the spring of 1985. The students were in the school of humanities and social science, and this was their first programming course. Although they went through 12 lessons with the LISP tutor, we only analyzed data from the first 6 lessons before their midterm. This midterm was a paper-and-pencil test and so provides us with an external validation of any results we get on performance with the LISP tutor. These lessons involved (a) introduction to some basic LISP functions; (b) introduction to how to define one's own functions; (c) conditionals and logical predicates; (d) helping functions; (e) input-output; and (f) iteration.

The typical history of a production is that it is introduced in a particular lesson and is involved in the coding of a number of problems. After that, it occurs irregularly and less frequently in later lessons. Depending on their centrality, different productions occur with rather different frequencies. This of course creates serious difficulties in trying to perform aggregate measure of performance over lessons.

There are some complications in defining the frequency of occurrence of a production. First, because alternative solutions to problems were possible, it was not guaranteed that the same productions would be used by all subjects. What appears as frequency is a measure of how often the production could have occurred. Some subjects do not contribute to some possible occurrences. This is not as serious a problem as one might think because a lot of the variation concerned changes in order of code or changes in choice of just one or two functions. Thus the variation in production use was much lower than the variation in number of distinct solutions produced.

The other complication is produced by remedial problems. When students are judged weak on certain productions, they are required to do remedial problems, which offers additional practice on these productions. Students need, on average, about 15% extra remedial problems, although there is large individual variation. We ignore remedial problems in the analyses that we report.

Thus our data in its finest grain can be broken down according to the dimensions of lesson, production, and opportunity for that production within the lesson. Crossed with these are the four dependent measures listed earlier—namely, time per production, time for correct production firings, probability of a correct production firing, and mean number of errors for a production firing.

RESULTS

I would like to present the majority of the data organized by lesson and aggregated over production. However, to explain this aggregation process and to get a better feel for the data, I believe it would be worthwhile to look at one lesson in more detail. Lesson 2 is appropriate for this purpose. Table 2.2 lists the productions we monitored. The first 9 were first introduced in Lesson 2 and the last 9 had been introduced in the previous lesson.

Fig. 2.3 and 2.4 plot the performance on the new productions for this lesson. Fig. 2.3 plots the times for correct use of the production (where the maximum value was set at 200 seconds), and Fig. 2.4 plots the mean number of errors, where this statistic has a maximum of three. We plotted just times for correct productions in order to get a measure that is independent of errors. As noted earlier, we also analyzed time aggregated over corrects and errors. This measure does not seem to reveal any additional insights. We choose to analyze total number of errors per opportunity in Fig. 2.4 rather than probability of error because we believe it is a better measure of student difficulty. Students often make single errors and correct them as slips. Every time an error cannot be corrected, it is further evidence that a student has a fundamental difficulty.

We have plotted these measures as a function of the times a production has been tested in the session. Both scales are logarithmic. Different productions occurred a different number of times, but we calculated a weighted average to provide our best estimate of how the mean changed with practice.[1] As can be seen, this mean shows a marked drop-off from first to second test and a very modest decline after that. The average improvement from first to second trial is almost 50% for time and over 50% for accuracy. We plot this on a log–log scale to make the point that this drop-off is not just part of the power-law improvement normally seen for a skill. Figures 2.5 and 2.6 plot the lesson averages for lessons

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[1] The average for the first opportunity is the average of the logarithms. The *n*th average, a_n, is calculated from *n-1*st average a_{n-1} as $a_{n-1} + i_n$ where i_n is the average change in the logarithm values of those productions for which there is both a *n-1*st and *n*th observation.

TABLE 2.2
Productions Monitored in Lesson 2

1. specify-function-name: Codes the symbol corresponding to the production name.
2. specify-function-params: Codes the parameters of the function.
3. code-nil: a production for coding the special symbol *nil.*
4. code-append: a production to generate the LISP combining function append.
5. code-reverse: a production to generate the LISP function reverse which codes the reverse of a list.
6. code-cosine: a production to code the LISP function *cosine.*
7. code-sine: a production to code the LISP function *sine.*
8. code-square: a production that codes the square of a number by taking the product of two numbers.
9. check-arg: a production that codes as an argument to a function called a parameter in the function definition.

The following productions were introduced in lesson 1 but reappear in lesson 2

10. code-car: a production to code the LISP function car that gets the first element of a list.
11. code-cdr: a production to code the LISP function *cdr* that gets the rest of a list.
12. code-second: a production that gets the second element of a list. This is coded as a *car-cdr* combination. This is treated separately because students have difficulty with precedence of unary operators.
13. code-cons: a production to code the LISP combiner *cons* that inserts its first argument in front of the list that is its second argument.
14. code-list: a production to code the LISP combiner *list* that wraps parentheses around its arguments.
15. code-divide: a production that codes the LISP function *quotient* that takes the quotient of two numbers.
16. code-difference: a production that codes the LISP function *difference* that subtracts its second argument from its first argument.
17. code-times: a production that codes the LISP function *times* that multiplies its arguments.
18. code-number: a production that codes a number argument to a function.

2, 3, and 5 and the average of these averages (lesson 1 measures are peculiar on the first trials because typically the teacher is coaching, and there are very few new productions introduced on lessons 4 and 6). It makes even clearer the point that the drop-off from the first to second trial is discontinuous.

Note that the rate of improvement after the first trial is basically linear in these two logarithmic measures. This implies a power function relating either time or errors to amount of practice. This is what is typically found in studies of practice. However, the clear discontinuity from trial 1 to trial 2 is something that has not been examined in detail until now. It is consistent with the knowledge compilation mechanism in ACT*, basically a one-trial learning mechanism. One might attribute the drop-off in errors to students just debugging their misconceptions from reading. Thus, it is significant that this discontinuity also shows up in times for errorless trials as well as number of errors.

Another question concerns how performance changes on productions across lessons. Figure 2.7 is an attempt to analyze this. We have plotted performance on

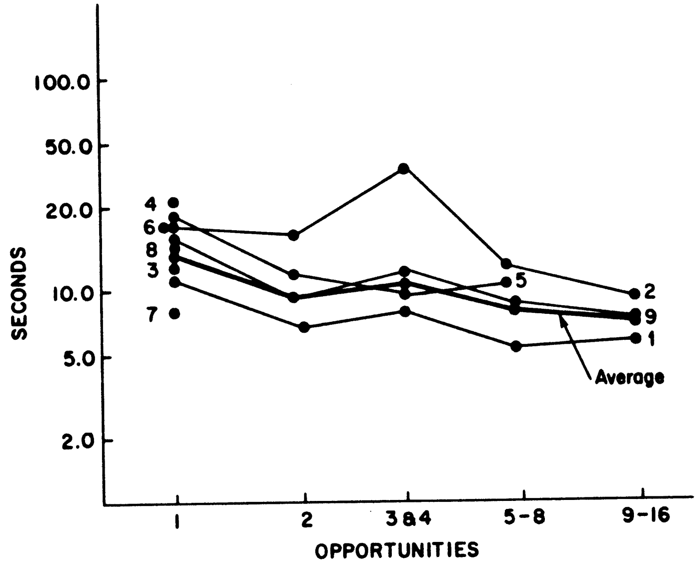

FIG. 2.3. Mean times for correct coding of the actions corresponding to 9 productions introduced in lesson 2. See text for explanation of the productions.

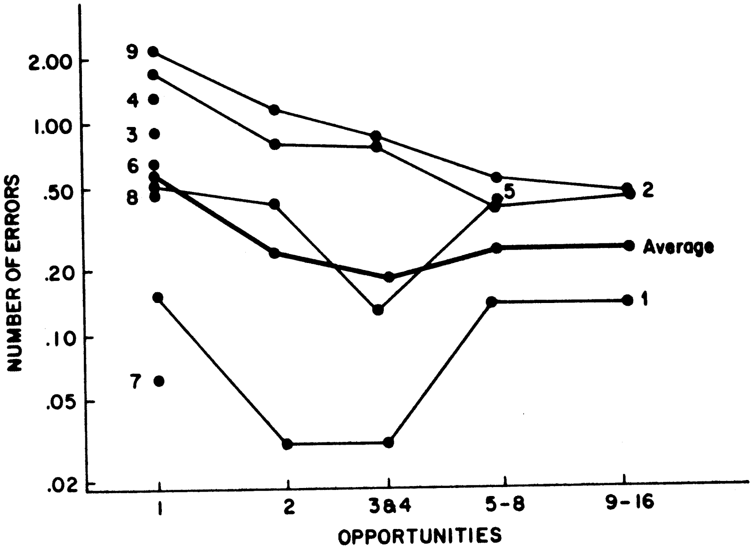

FIG. 2.4. Mean number of errors in coding the actions corresponding to 9 productions introduced in lesson 2. See text for an explanation of the productions.

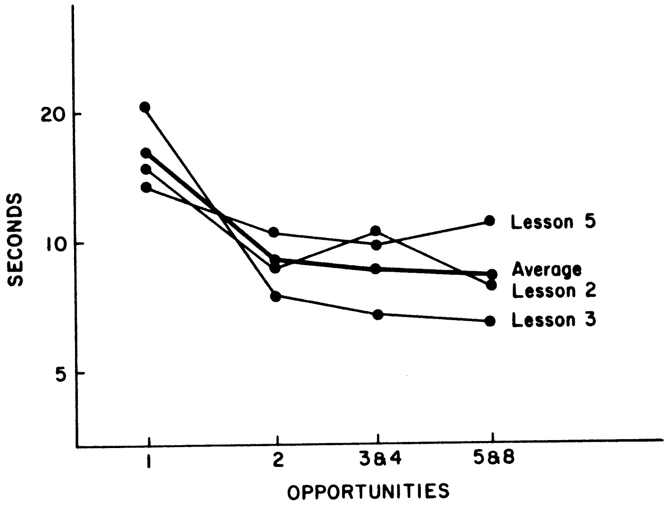

FIG. 2.5. Average times for coding actions corresponding to new productions in lessons 2, 3, and 5.

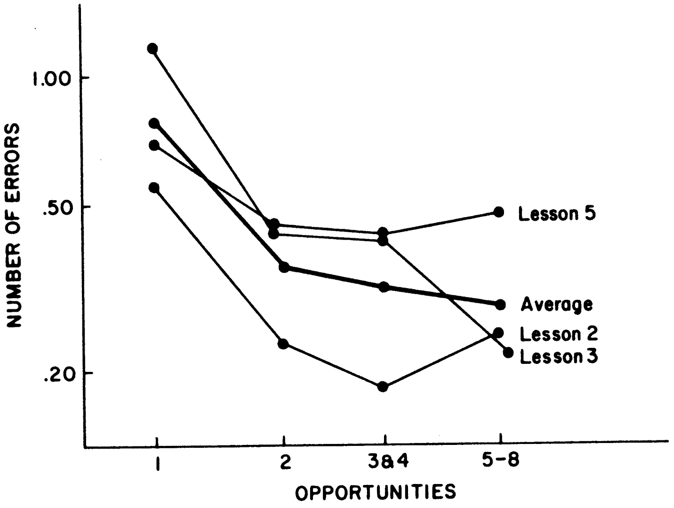

FIG. 2.6. Average number of errors in coding actions corresponding to new productions on lessons 2, 3, and 5.

last occurrence in lesson *n*-1, first performance in lesson *n*, and last performance in lesson *n*. Figure 2.7 plots these patterns for the lesson pairs 1 and 2, 2 and 3, 3 and 4, and 5 and 6 (there are few shared productions between 4 and 5). The average of these lesson averages makes the pattern even more apparent. There is perhaps a little forgetting between lessons but considerable improvement within lessons. This is more apparent with the accuracy measure than the time measure. Although they are not exactly the same productions plotted in each curve, note that the curves tend to get lower across lessons, also consistent with a gradual improvement with practice.

One might wonder how much support these analyses offer for the existence of the production rules assumed by the LISP tutor. The apparent regularity of the data is consistent with the view that the LISP tutor provides the psychologically correct decomposition of the skill. However, these production rules do tend to correspond to pieces of code in LISP. For instance, *code-car* corresponds to typing *car* and *check-arg* corresponds to typing a variable name. What if we simply monitored how accurately students wrote these pieces of code and ignored the production-rule analysis? Although correlated, it is not the case that a code-based analysis is identical to a production-rule analyses. This is because in some cases there is a many-to-one relationship between production rules and types of LISP code. For instance, although *code-car* usually is responsible for generating *car*, there is a special production, *code-second*, that corresponds to *car* when we

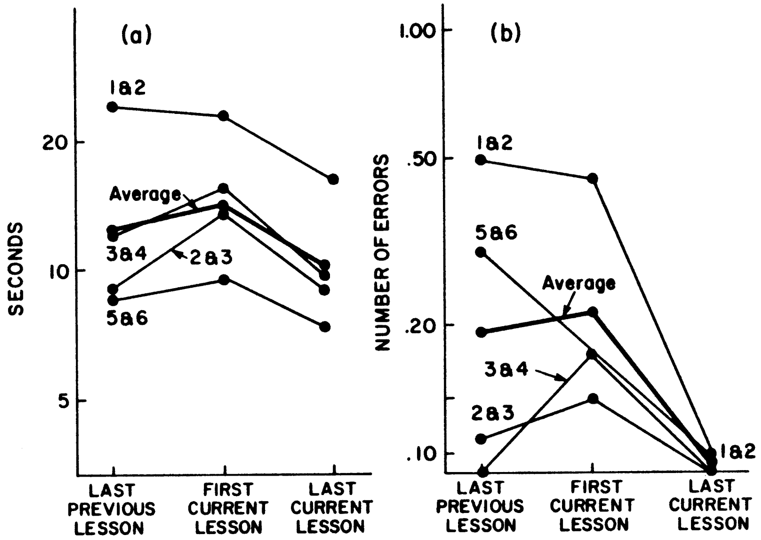

FIG. 2.7. Average times and errors for old productions in a particular lesson. Plotted is a measure for last opportunity in previous lesson, first opportunity in current lesson, and last opportunity in current lesson.

are composing a *car-cdr* sequence. The first time subjects typed *car* as *code-second* in lesson 1 they made .68 errors. We can go back to their error rate on the preceding *code-car* and find it was just .41. This increase from .41 to .68 deviates from the general improvement within a lesson. All other comparisons we looked at show the same trend. Coding *cond* to terminate an iteration is different than coding *cond* generally. Subject make 1.05 errors in the *cond* in iteration compared to .32 errors for the previous *cond*. Coding a number to initialize a variable is different than passing it as an argument to a function. Subjects make 1.64 errors on their first use of a number to initialize compared to .24 errors in the previous use of a number. Using *setq* to initialize a local variable is controlled by a different production than the one that uses *setq* to initialize a global variable. Subjects make .89 errors on their first local variable *setq* compared to .29 for their previous global variable *setq*. Coding a variable that is a function parameter is governed by a different production than the one that codes a global variable. The difference in error rate is 2.18 versus .38. Coding a parameter is different in turn from coding a local variable in a function. By the time subjects get to local variables, their error rate for parameters has dropped from 2.18 to .03. Their error rate in the first local variable is .84. The upshot of all these comparisons is that subject improvement is better defined in terms of the LISP tutor productions than surface code. The regularity of the data really is evidence for the LISP Tutor's production-rule analysis.

The overall pattern of data displayed in these graphs is consistent with the ACT* learning mechanisms. There is the discontinuous point of learning from first trial to second due to knowledge compilation, a power-law growth in strength (and hence performance measure) with practice, and some forgetting (loss of strength) between lessons. It should also be noted that, with the exception of the first trial discontinuity, we find nothing in LISP learning that would be surprising from the results of verbal learning.

INTER-PRODUCTION CORRELATIONS

Another question of interest is how well does performance on one production correlate with performance on another production. We looked at the measures of number of errors in calculating these correlations because they prove to yield the largest correlations. In one analysis we looked at patterns of correlation between lessons 1 and 2, 2 and 3, and 3 and 4. We calculated how well a production from one lesson correlated with itself on another lesson. Basically, this involves getting mean number of errors made on a production for each lesson for each subject and looking for a correlation over subjects for each production.

The average correlation of all the productions that repeated across trials was .26. This might not seem very large, but because most productions only occur a few times in a lesson and the individual measures are inherently quite noisy, correlations will be low. Under one model the expected maximum correlation is only .33.[2]

We gathered two other correlation measures between lessons. First, we broke the productions on individual lessons into those that deal with list operations and those that do not. Thus, we divided the productions into disjoint sets. We looked at correlations within sets (excluding correlations between the same production reported in the preceding paragraph) and between sets. The average correlation was .17 both within and between sets. These numbers are significantly lower than the correlation between the same production on both lessons, and obviously the two correlations were not significantly different. Intuitively, this is surprising because it indicates that there is no tendency for productions of the same type to cluster. Both correlations are quite significantly different than zero, of course, indicating some systematic individual differences among subjects.

We also calculated correlations among productions within the same lesson, excluding correlation of a production with itself, which would be 1. The average within-lesson correlation is .23, which is significantly higher than the between-lesson correlation and does not significantly differ from the between-lesson, same-production correlation. This indicates some tendency for subjects to have good lessons or bad lessons, which is not surprising. Again, if we break up these within-lesson correlations into correlations within list and non-list productions and correlations between list and non-list productions, there is no difference.

The failure to find evidence for a clustering among productions involved with list operations is surprising. We had strongly suspected that some subjects would do well on all list operations and some would not. Although this is in fact the case, it appears that the interproduction correlations are not higher than found between apparently unrelated productions. Thus, there appears to be a general ability factor, but not one associated with list operations.

One possible conclusion is that productions do not break up into thematic clusters, but it is possible that we simply did not intuit properly the factors that would cause productions to cluster. To see if this was the case, we subjected the data from the six lessons to factor analyses. We took the matrix of subject-by-production error means for each lesson and submitted it to a standard factor-analysis program. We then looked at the first two factors extracted for each lesson. We have looked at the other three performance measures, but error totals

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[2] We have on average about four observations per production per lesson. Assume that each observation is a binomial with a probability p of generating an error. Assume that for half the production $p = .33$ and for the other half $p = .67$. The correlation expected between number of errors for four observations on day 1 and four observations on day 2 is .33.

consistently show largest variance accounted for in the first two factors. The data from the lessons structured as follows.

1. Number of Productions involved in the factor analysis. In all cases there are 34 subjects.
2. Variance accounted for by first factor, second factor, and third factor. The variance accounted for by the second factor indicates what we gain by including it, and the variance accounted for by the third factor indicates what we lose by excluding it.
3. Which productions loaded most heavily on each factor after rotation. If a production loads heavily on both, we report the factor it loads most heavily on. We exclude productions with a factor loading of less than + .6. We report the magnitude of the loadings in parentheses.

Lesson 1—15 Productions:

Factor 1	Factor 2	Factor 3
41% of the variance	13% of the variance	10% of the variance
global variables (.81)*	quotient (.74)*	
list function (.79)*	formula for square (.61)*	
cdr function (.77)*		
coding a number (.76)*		
first arg to setq (.75)*		
cons function (.73)*		
setq function (.72)*		
quoted constant (.63)*		
plan for second element in a list (.61)*		

Lesson 2—19 Productions:

Factor 1	Factor 2	Factor 3
34% of the variance	12% of the variance	8% of the variance
coding a number (.83)	reverse function (.81)*	
cdr function (.81)	coding parameters in function definition (.72)*	
coding function name (.78)*	coding NIL (.71)*	
difference function (.68)	cons function (.61)	
quotient function (.65)		
formula for square (.64)		
defun function (.62)*		

Lesson 3—31 Productions:

Factor 1	Factor 2	Factor 3
28% of the variance	11% of the variance	10% of the variance
coding a number (.86)	coding an else (*t*) clause (.71)*	
cond function (.82)*	coding a parameter (.68)	
coding a constant (.78)	member function (.65)*	
lessp function (.75)*	reverse function (.64)	
cons function (.72)	or function (.63)*	
null function (.70)*	not function (.62)*	
global variable (.62)		

Lesson 4—18 Productions:

Factor 1	Factor 2	Factor 3
28% of the variance	12% of the variance	10% of the variance
equal function (.79)	case within a cond (.68)	
car function (.74)	coding an else (*t*) clause (.66)	
last function (.74)	coding a parameter (.62)	
cdr function (.69)		

Lesson 5—28 Productions:

Factor 1	Factor 2	Factor 3
45% of the variance	10% of the variance	6% of the variance
Prog function (.90)*	code a local variable (.83)*	
numberp function (.80)*	print function (.74)*	
case within a cond (.77)	coding a parameter (.72)	
coding a loop tag (.73)	plus function (.69)	
not function (.73)	cond function (.69)	
go function (.72)	read function (.69)	
resetting a variable (.67)*	coding the variable to be reset (.68)*	
coding a constant (.67)	list function (.67)	
return function (.64)	difference function (.61)	
initializing a variable (.62)*		

Lesson 6—27 Productions:

Factor 1	Factor 2	Factor 3
29% of the variance	11% of the variance	7% of the variance
code a local variable (.88)	code an initial value for iteration (.83)*	
coding a parameter (.81)	initialize a variable for iteration (.83)*	
coding a loop tag (.76)	prog function (.76)	
equal function (.76)		
coding repeat tag (.76)		
coding a variable (.68)		
plus function (.68)		
coding a number (.63)		
difference function (.62)		
greaterp function (.61)		

WHAT DOES IT MEAN?

We could not make a great deal of sense out of this pattern. Productions were not apparently clustering according to any semantic feature. In an attempt to make sense of this we took each subject's factor scores for the two factors for each lesson and thus got twelve measures for each subject. We subjected these to a factor analysis to determine which lesson factors would cluster together. The first meta-factor extracted accounted for 36% of the variance and the second meta-factor 16% of the variance. The third meta-factor (which we will ignore) accounted for 14% of the variance.

The following factors loaded on the first meta-factor—factor 2, lesson 1; factor 2, lesson 2, factor 2, lesson 3; factors 1 and 2, lesson 5; and factor 2, lesson 6. The following factors loaded on the second meta-factor—factor 1, lesson 3; factors 1 and 2, lesson 4; and factor 1, lesson 6. Factor 1 from lesson 1 and factor 1 from lesson 2 did not load on either meta-factor, suggesting that these may reflect peculiar start-up features in dealing with LISP.

It only became apparent after considerable inspection what unites the factors categorized together under each meta-factor. Twenty-two of the 34 productions organized under meta-factor 1 were introduced in that lesson, whereas only 3 of the 23 productions organized under meta-factor 2. The new productions are starred in the listings just given. Thus the first factor is basically an acquisition factor because it reflects performance in productions being acquired in that lesson, whereas the second is a retention factor because it reflects performance in productions presumably already acquired in previous lessons, and, thus, deficits must be due to forgetting. This helps explain why the actual clustering of productions seemed arbitrary semantically and why the clusters do not stay constant across lessons. The second factor correlates .62 with math SAT; the first factor only correlates .03. Neither factor correlates with verbal SAT or grade-point average at Carnegie Mellon.

It seemed worthwhile to see how these categories did at predicting performance on paper-and-pencil tests. We used the midterm and final exam tests of the 34 students. The midterm was administered right after completing lesson 6, whereas the final was administered after 6 more lessons with the tutor. We have not had the opportunity to analyze the data from these 6 lessons. We took subjects' factor scores on these two meta-factors and classified them into above or below the median. This gave us ten subjects in both the high–high and low–low categories and seven in both the high–low and low–high categories.

Table 2.3 presents the data from the midterm exams in terms of scores out of 24, and Table 2.4 presents the final grades out of 28 similarly classified. There are significant effects of both factors on midterm grades—$F(1,30)=5.1$ for acquisition and $F(1,30)=6.4$ for retention. The interaction is not significant—$F(1,30)=2.8$. Again, only the main effects were significant on final exam–acquisition factor with $F(1,30)=6.3$ and retention factor with $F(1,30)=9.7$

TABLE 2.3
Midterm Grades

	Low Acquisition Factor	*High Acquisition Factor*
Low Retention Factor	3.8 ± .9	9.7 ± 1.3
High Retention Factor	10.1 ± 1.9	11.0 ± 1.5

TABLE 2.4
Final Exam

	Low Acquisition Factor	*High Acquisition Factor*
Low Retention Factor	9.8 ± 1.7	13.4 ± 1
High Retention Factor	14.2 ± .9	17.8 ± 1.9

CONCLUSIONS

In my opinion this analysis of student behavior is marvelous for the lawfulness and simplicity of the picture it paints. It is particularly consistent with the ACT* production-system analysis of skill acquisition. The behavior is quite regular when analyzed with respect to the production rules used in the LISP tutor. It shows evidence for a one-trial learning episode consistent with knowledge compilation and a more gradual learning consistent with the power-law strengthening process.

Productions are independent and modular, as would be predicted by the ACT* theory. The only production rules with which a particular production correlates especially strongly is with itself. This is consistent with the notion that each production is learned independently. In particular, we found no evidence that productions were being clustered because some subjects were having difficulty with list manipulations. Some productions are clearly more difficult than others and some subjects are clearly more capable than others, but there does not appear to be any interaction between the two factors. Subjects' overall ability impacts on how fact they learn all aspects of LISP, but except through this general ability factor the learning of one production is independent of the learning of others.

When we did an atheoretical factor analysis looking for evidence of an interaction, the only thing we found were two somewhat independent general-ability factors of acquisition and retention on which subjects could be sorted and on which old and new productions could be sorted. This result clearly does not compromise the basic cognitive assumption of independence of productions, although it raises some interesting questions about what the real nature of these two ability factors might be.

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