



Using Cognitive Modeling to Understand the Roles of Prefrontal and Posterior Parietal Cortex in Algebra Problem Solving

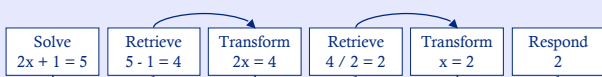
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Introduction

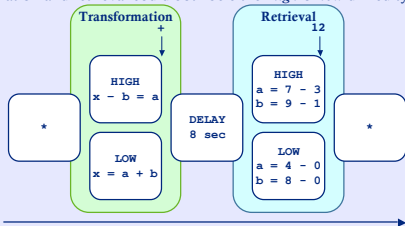
- Retrieval and transformation are highly correlated in algebra problem solving:



- The current instantiation of the ACT-R theory (Anderson, 2005) associates retrieval and transformation with activity in prefrontal and posterior parietal cortex, respectively
- Activity in these regions are likewise correlated during naturalistic problem solving (Anderson et al., 2003)
- Can we isolate the processes of transformation and retrieval in algebra problem solving and manipulate them independently?
- If so, can we also isolate activity in their neural correlates?

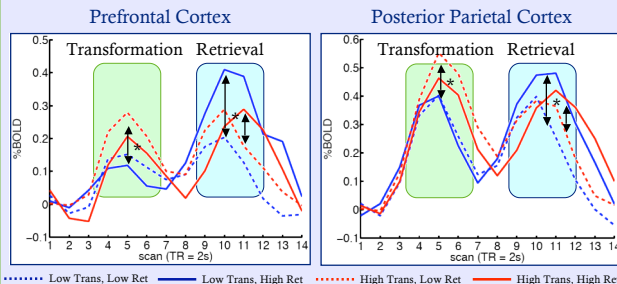
Method

- 20 participants run in a BOLD fMRI study
- Algebra equations were created that needed to be solved in 2 phases:
 - Transformation phase: isolate x from a and b
 - Retrievals are delayed until numerical values of a and b are given
 - Retrieval phase: calculate the value of x given the numerical values of a and b
- The transformation and retrieval could both be either high or low difficulty

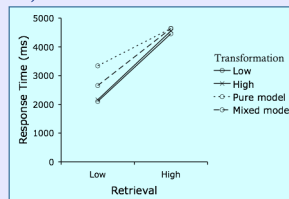
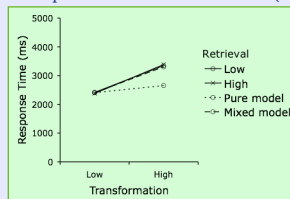


Results and Modeling

- Manipulating retrieval load of algebra problems results in differential activity in both prefrontal and posterior parietal cortex
- Manipulating transformational requirements of algebra problems results in differential activity in both prefrontal and posterior parietal cortex



- Why did our manipulations fail to isolate the activity in these regions?
 - Explanation 1: These regions are not functionally distinct as characterized by ACT-R
 - Explanation 2: Our task manipulations failed to properly isolate retrieval and transformation
- We designed two ACT-R models to determine the plausibility of Explanation 2
 - Pure Model: Encompassed our initial assumptions about the design but failed to fit the RT data ($R^2 = .50$)
 - Mixed Model: Assumed that both manipulations had retrieval and transformation components and fit the RT data well ($R^2 = .94$)



Pure Model

Low Transformation		High Transformation	
Low Retrieval	High Retrieval	Low Retrieval	High Retrieval
Encode $x + a = b$	Encode $x + a = b$	Encode $x - b = a$	Encode $x + a = b$
Retrieve key for +	Retrieve key for +	Transform Retrieve key for +	Transform Retrieve key for +
Respond +	Respond +	Respond +	Respond +
Encode $a = 4 - 0$	Encode $a = 7 - 3$	Encode $a = 4 - 0$	Encode $a = 7 - 3$
Encode $b = 8 - 0$	Retrieve $7 - 3 = 4$	Encode $b = 8 - 0$	Retrieve $7 - 3 = 4$
Retrieve $4 + 8 = 12$	Retrieve $b = 9 - 1$	Retrieve $4 + 8 = 12$	Encode $b = 9 - 1$
Retrieve key for 2	Retrieve $9 - 1 = 8$	Retrieve key for 2	Retrieve $9 - 1 = 8$
Respond 2	Retrieve $4 + 8 = 12$	Respond 2	Retrieve $4 + 8 = 12$
Retrieve key for 2	Respond 2	Retrieve key for 2	Respond 2

Mixed Model

Low Transformation		High Transformation	
Low Retrieval	High Retrieval	Low Retrieval	High Retrieval
Encode $x + a = b$	Encode $x + a = b$	Encode $x - b = a$	Encode $x + a = b$
Retrieve key for +	Retrieve key for +	Retrieve Operator Retrieve key for +	Retrieve Operator Retrieve key for +
Respond +	Respond +	Transform Retrieve key for +	Transform Retrieve key for +
Encode $a = 4 - 0$	Encode $a = 7 - 3$	Encode $a = 4 - 0$	Encode $a = 7 - 3$
Encode $b = 8 - 0$	Retrieve $7 - 3 = 4$	Respond +	Respond +
Retrieve $4 + 8 = 12$	Encode $b = 9 - 1$	Encode $a = 4 - 0$	Encode $a = 7 - 3$
Retrieve key for 2	Retrieve $9 - 1 = 8$	Retrieve $4 + 8 = 12$	Retrieve $7 - 3 = 4$
Respond 2	Respond 2	Retrieve key for 2	Encode $b = 9 - 1$
Retrieve key for 2	Retrieve $4 + 8 = 12$	Respond 2	Retrieve $9 - 1 = 8$
Retrieve key for 2	Respond 2	Retrieve key for 2	Retrieve $4 + 8 = 12$
Respond 2	Respond 2	Retrieve key for 2	Respond 2

Conclusions

- There is a lot of difficulty isolating the basic cognitive processes characterized by ACT-R
- The difference between the roles of prefrontal and posterior parietal cortices in algebra problem solving remains unclear
- Using cognitive modeling can help in the interpretation of behavioral and neuroimaging data

References

• Anderson, J.R., Qin, Y., Sohn, M.-H., Stenger, V.A., Carter, C.S. (2003). An information-processing model of the BOLD response in symbol manipulation tasks. *Psychonomic Bulletin & Review*, 10, 241-261.

• Anderson, J.R. (2005). Human symbol manipulation within an integrated cognitive architecture. *Cognitive Science*, 29, 313-341.

• Danker, J.F., Anderson, J.R. (2007). The roles of prefrontal and posterior parietal cortex in algebra problem solving: A case of using cognitive modeling to inform neuroimaging data. *NeuroImage*, 35, 1365-1377.