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An ACT-R Predictive Model of Performance

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Abstract

The ACT-R computational modeling architecture has demonstrated the ability to model both recency and frequency effects in memory with much success (e.g. Anderon & Lebiere, 1998); and through the incorporation of new decay parameters at each data point, has also been shown to capture the spacing effect (Pavlik & Anderson, 2003, 2005). Stemming from the aforementioned literature, the current research sought to build an equation capable of handling the prediction of performance at later, distributed points in time, thereby breaking from the tradition of post-fitting data. As such, we integrated a single activation-based decay rate into the ACT-R General Performance Equation (Anderson & Schunn, 2000), and scaled predictions by amount of training history improvement. We tested this algorithm by extrapolating learner knowledge states from initial points in data, and predicting performance at later points in time, across different intervals of time. Implications are discussed.

Introduction

Although more than a century of published research in the learning and forgetting of knowledge and skill has been amassed (Ebbinghaus, 1885), consensus for the cognitive mechanisms responsible for a learners' enhanced retention as a function of increased temporal spacing of practice sessions has yet to be reached (e.g. Crowder, 1976, Landauer, 1967, Madigan, 1969, Glenberg, 1979). As theorists are not unified in how or where memory traces are stored to help explain the spacing effect, it is not surprising that computational cognitive process models have had difficulty implementing parameters to simulate and capture these complex human performance curves. Only quite recently (Pavlik & Anderson, 2003; 2005) have strides been made to mathematically simulate and post-fit these effects.

The approach to this research was to assess existing computational models of learning and forgetting, build upon existing strengths, and develop new techniques to circumvent existing weaknesses.

Models

In this portion, we lay out prior computational models that provided the basis for this work, and finally present the modified algorithm.

ACT-R General Performance Equation (Anderson & Schunn, 2000)

The following equation provides the basis for our prescriptive and predictive modifications. On a basic level, it is a derivative of ACT-R equations and encapsulates the power law of practice, the power law of forgetting, and the multiplicative effect of practice and retention (relation between amount of practice and duration of time for which knowledge must be maintained). This corresponds with traditional ACT-R theory which suggests that neural degradation chips away at existing memories over time. The General Performance Equation is formally expressed by:

$$\boldsymbol{A \cdot N^{c} \cdot T^{-d}}$$

where A is a scalar, N is amount of practice, c is the rate of learning, T is time since learning, and d is the decay parameter. The collective effect of this algorithm is that performance continues to improve as amount of practice increases, and continues to degrade as time between learning and retention increases. Preservation of knowledge then depends upon leveraging the amount of practice against the time between repetitions.

To emphasize the reasons for utilizing its core components in our modified equation, we first demonstrate the strengths this model possesses. Indeed, this equation has fits many varieties of data sets quite well, including studies concerning knowledge retention, knowledge acquisition, skill retention, and skill acquisition (see Figures 1-4, respectively).

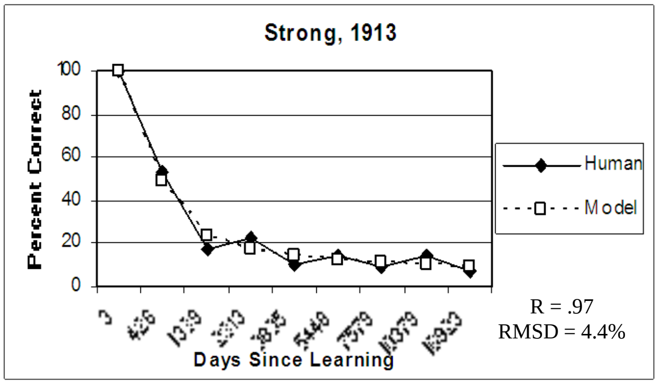


Figure 1: Model fit to knowledge retention.

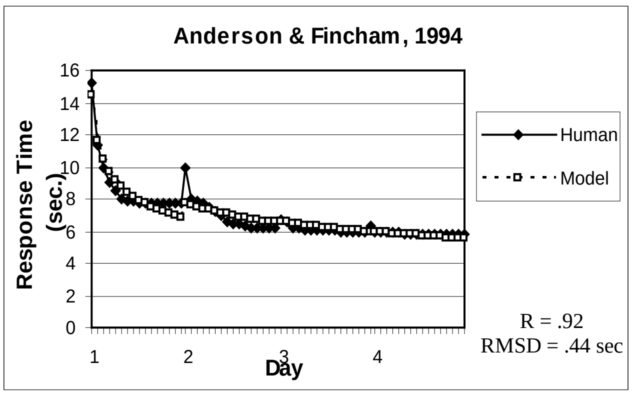


Figure 2: Model fit to knowledge acquisition.

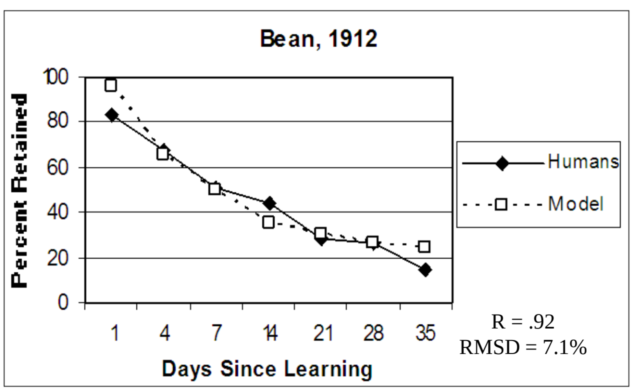


Figure 3: Model fit to skill retention.

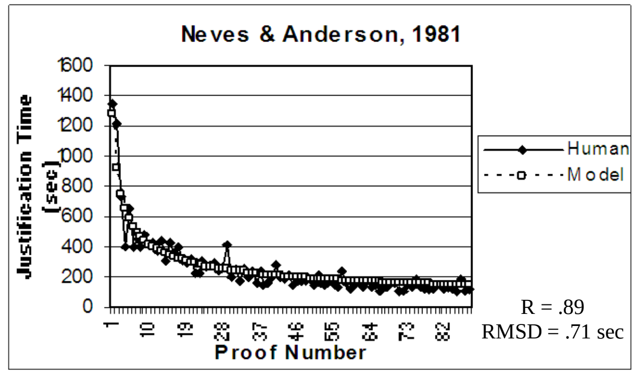


Figure 4: Model fit to skill acquisition.

Clearly, these figures demonstrate the usefulness of the General Performance Equation for many types of data sets, and provide correlation coefficients of .89 to .97 for fits with human performance. We now turn to a dimension of learning and forgetting that this equation does not handle well, namely, the spacing effect.

Mathematical Weaknesses of the General Performance Equation for Handling the Spacing Effect Human performance studies have revealed that learning and forgetting do not continuously improve or degrade over extended periods of time, but rather they approach asymptote. For example, an item presented at longer intervals of time will be retained better than an item crammed more tightly together in temporal space. The practice function in its current form would assume a discrete increment to be added at each presentation time of the item, and would necessitate a greater decay rate to be incorporated for an item presented across greater intervals of time. This would result in the equation modeling better performance for crammed study than distributed study, and thus deviates from what literature has shown human performance to be. With regard to forgetting rates, Woodworth (1938) aptly described this scenario: “If two associations are now of equal strength but of different ages, the older one will lose strength more slowly with the further passage of time.” Again, the General Performance Equation is not equipped to handle this type of phenomenon, and in its current form would instead produce the converse effect, such that greater passages of time would incur greater decay rates in the model. Thus, as demonstrated in Figure 5 below, the General Performance Equation clearly loses its ability to model human performance when distributed training regimens are a part of the procedure, and correlation between the model and the human drops to .49.

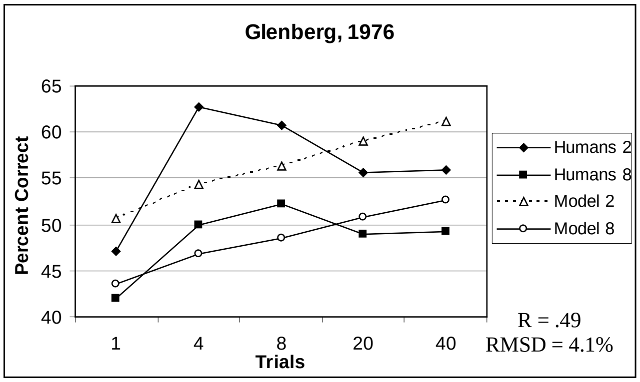


Figure 5: General Performance Equation Model fits to data spaced at practice intervals of every 2 and every 8 trials.

Activation-Based Model of the Spacing Effect (Pavlik & Anderson, 2003; 2005)

The following equation incorporates some useful concepts that capture effects of distributed practice in human performance. Namely, new rates of decay are computed based on current activation levels for each memory trace. This in effect, reflects the overall mass of practice, such that highly activated traces result in faster rates of decay than less highly activated traces. Theoretical credence is given to this type of mechanism on a neurological level, such that very high synaptic stimulation (corresponding to high activation levels of a trace in the model) impedes information transfer due to limitations in biology (Scharf, Woo, Lattal, Young, Nguyen, & Abel, 2002). Thus, at the neurological level, massed learning is not as effective as distributed learning. This equation is formalized by the following:

$$A \cdot N^{c} \cdot T(h)^{m_n(t_{1\ldots n}) = \ln\left(\sum_{j=1}^{n} t_j^{-d_i}\right)}$$

where parameters appearing in the General Performance Equation are defined the same in this equation; but additionally, this activation-based model incorporates an interference scalar, *h*, to decrease objective time since learning and produce better model fits, and modifies the decay rate such that *m* is the activation of the item *i*, t_j is how long ago practice of the item occurred, and *n* is the number of trials provided to practice the item. In essence, this results in a new decay rate being calculated at each data point, and provides the equation with great power and flexibility for fitting human data, as seen in Figure 6. In contrast to the General Performance Equation which produced a correlation coefficient of .49, this activation-based model increases the correlation coefficient to .98.

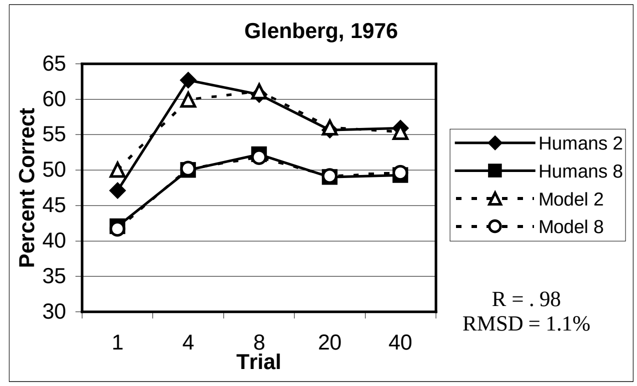


Figure 6: Activation-Based Model fits to data spaced at practice intervals of every 2 and every 8 trials.

Weakness of the Activation-Based Model of the Spacing Effect Although this model has the ability to post-fit human performance with great accuracy, the question of over-fitting the data set must be raised. In order to predict performance at future points in time, one must have precise knowledge of activation levels amassed over training history up to that point. As this model contributed greatly to the literature through its ability to account for the spacing effect mathematically, it lacks the capability to predict performance, and it may be less flexible in generalizing to other data sets without massive manipulation of each parameter.

Proposed Predictive Model

Building upon strengths of the previous equations, we sought to formalize an algorithm to capture recency, frequency, and spacing effects; while also providing flexibility and capability for predicting performance at later points in time. This equation is formalized by the following:

$$S \cdot N^{c} \cdot T^{-a}$$

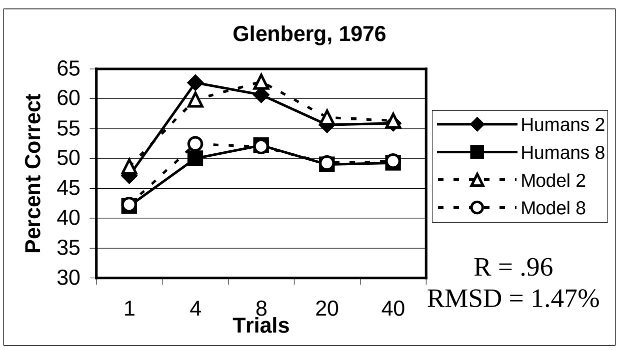
where *S* equals the original scalar (*A* in the General Performance Equation) multiplied by training history (known improvement rate between initial time of learning and last known retention session), and *a* equals an activation-based decay parameter that enfolds an exponential function into the decay rate, such that:

$$a = d \cdot e^{(m-1)} + d(intercept)$$

To further elaborate the activation-based decay parameter *a*, *m* equals the activation level at the latest known data point, defined by ln T^{-d}., so that this parameter is calculated from known training history and is based upon the original decay rate and activation level at the latest known point.

This equation differs from the Activation-Based Model in two key ways. First, this model does not include an interference scalar to modify the length of time passing between retention trials in order to produce better fits. We believe the amount of time between trials should remain true to how much time actually goes by. Second, we do not produce a new decay parameter for every point in the data set. Rather, we formulate one modified decay rate at the end of known training history, and that new decay rate accounts for the amount of practice accrued under the initial decay rate, and bases future degradation of knowledge upon the last known level of activation. Thus, rather than reformulating a new decay rate for each and every data point, we recalculate a new decay rate at the last point only, and make predictions for future performance rather than post-fitting the models' data to match complex human performance curves.

Ability to Account for Spacing Effect In order to demonstrate the efficacy of our Predictive Model against both the General Performance Equation and the Activation-Based Model, we plotted our model fits to the same data set. Figure 7 reveals correlations of .96 between our model and the data, showing a marked improvement over the General Performance Equation (.49) and competitiveness with the Activation-Based Model (.98) using fewer free parameters.



Method We utilized existing data sets as a testbed, and divided the data into sections of varying length to test predictive value of the equation. We sought to first trace knowledge and assess training history by optimizing the fit of our model to initial portions of the data set. When that formula had been attained, we then multiplied the original scalar by the rate of improvement, and modified the decay parameter to account for the current activation level at the last known data point. We then applied the Predictive Model to predict performance at different intervals down the line and compare those predictions with human data.

Results Firstly, we optimized the model fit for Missions 1-5 using CERTT's data set of team performance (Cooke, 2005). Missions 1-5 were practiced over the course of one day, and each mission lasted approximately 40 minutes long. We then redefined our decay parameter using the activation level at Mission 5, and predicted performance for Missions 6, 7, and 8. These final three missions were tested over the course of one day at an interval 10-14 weeks later. Again, each mission lasted 40 minutes long. Correlation between model and actual performance over Missions 1-5 was .97, and correlation between predicted performance and actual performance for Missions 6-8 was .96 (see Figure 7).

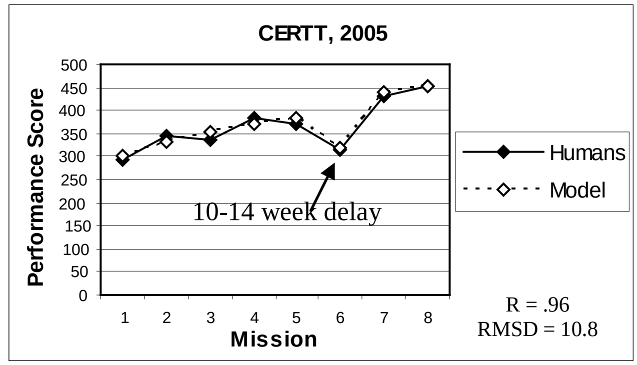


Figure 7: Predictive Model fits optimized to Missions 1-5 and predicted for Missions 6-8.

Resolution of Data As predictions are based upon the learner's training history, the predictive ability of this equation becomes more and more refined with greater amounts of practice history. In this sense, the equation has more of the learners' knowledge states to trace, and can make better predictions for how that knowledge state will change across time or with repeated exposures. To demonstrate the differences training history makes in predictive ability, refer to Figures 8 below for correlations between human and model predictions dependent upon the amount of data points available.

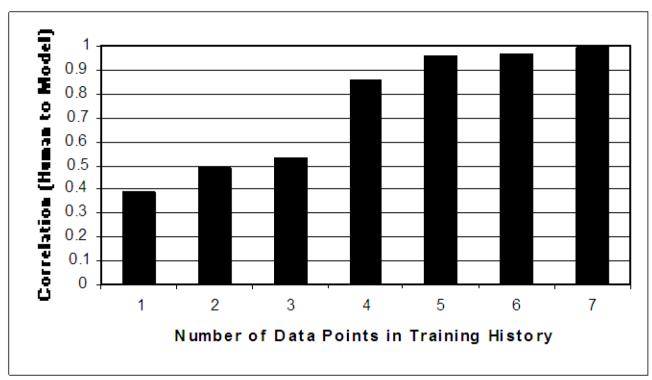


Figure 8: Correlations of Predictive Model and human performance based on amount of training history.

Discussion

The main contribution of this equation over its predecessors is its ability to predict performance at later points in time, even when practice is distributed at different intervals. This diverges from ACT-R and ACT-R derivative models that have simply post-fit model data to simulate human performance curves. Additionally, the equation itself was able to capture the spacing effect with optimized parameters more parsimoniously than prior work (Pavlik & Anderson, 2003; 2005).

As laid out by Pitt, Myung, & Zhang (2002), criteria for evaluating or selecting one computational model over another include (a) plausibility (are assumptions biologically or psychologically plausible?); (b) explanatory adequacy (is the theoretical basis reasonable?); (c) interpretability (do parameters make sense?); (d) descriptive adequacy (does the model describe observed data well?); (e) generalizability (can the model predict future performance?); and (f) complexity (is the model written in the simplest way to adequately capture the data?). As per these requirements, we believe that our model outperforms prior models with respect to plausibility (we removed the alteration of scaled time and interference scalars), generalizability (this model can make predictions for future performance), and complexity (model parameters are more parsimonious than prior work); while staying true to ACT-R neural, biological, and cognitive theoretical assumptions.

We do believe however, that further work will be needed to test the predictive capability of this equation across a greater number of data sets that encompass different variations of time intervals. Additionally, we would like to investigate differences between mathematical regularities of aggregate versus individual level performance, and assess how well our equation can account for likelihood of success at different resolutions of the data. Implications for this research could extend to predicting performance in educational or military domains for instance.

Acknowledgments

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