

# What role do cognitive architectures play in intelligent tutoring systems?<sup>1</sup>

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Running Head: Cognitive Architectures and Education

In his “A Bridge Too Far” paper Bruer (1998) suggests that cognitive psychology serves as an “island” to link research on the brain with research on instruction. He argues that a bridge going all the way from brain to instruction is impossible without this intermediate point to interpret the results from brain research and determine their implications for instruction. This paper is concerned with the issue of how one can bridge from basic cognitive psychology to education. A great deal of basic research and theory in cognitive psychology studies behavior in small tasks isolated from one another. Cognitive psychologists study how subjects recognize symbols, memorize lists of items, reason about syllogisms, process syntactic ambiguity, etc. Much of recent research in cognitive neuroscience has looked for brain correlates of such simple tasks. If there is going to be a bridge between cognitive psychology and education it is going to have to provide a bridge between such simple tasks and the much more complex cognition that occurs in the classroom.

Unfortunately, there is a large gap between such laboratory research in cognitive psychology and education. As Newell (1973) lamented over a quarter of a century ago, such research has failed to provide a characterization that integrates human cognition. Such an integration is absolutely essential for educational

applications. In learning geometry, for instance, students have to simultaneously deal with recognizing symbols, memorizing new information, processing the syntax of instruction, reasoning about the material, and much more. While he was not specifically concerned with educational applications, Newell introduced cognitive architectures as a solution to the problem of integration. Cognitive architectures are computational systems that try to characterize how the different aspects of the human system are integrated to achieve coherent thought.

This paper will be concerned with the implications of the ACT architecture for education. It will start with a description of the ACT\* architecture (Anderson, 1983) and how this served as the basis for a generation of intelligent tutoring systems (e.g., Anderson, Corbett, Koedinger, & Pelletier, 1995). Then it will turn to discussing the factors that motivated the development of the ACT-R architecture (Anderson & Lebiere, 1998). ACT-R is an architecture that analyzes cognition at the grain size which characterizes much of laboratory research and also is capable of putting these pieces together in a model of complex cognition. Thus, it provides the potential bridge between basic cognitive psychology and education.

We will first review the ACT theory and its connection with education through the construction of cognitive tutors. The key assumption of that theory is that complex cognition can be decomposed into simpler units. ACT-R currently analyzes cognition at a much finer grain size than past versions of the ACT theory and we will consider the consequences of this fine grain size for tutoring. We will show that it implies that it is important to monitor students at a much more detailed level than has been typical of our past tutors. Using eye movement studies we will show that there are indeed instructional advantages to be obtained by what we call high-density sensing of the student. We will end the paper with an assessment of whether it is possible to build one bridge that goes all the way from the simple tasks of cognitive psychology to instruction, or whether more bridges are needed.

## **The ACT Theory and Cognitive Tutors**

The basic assumption in all the ACT theories is that human cognition emerges through an interaction between a procedural memory and a declarative memory. Interestingly, this procedural-declarative distinction is finding increasing support from cognitive neuroscience with procedural learning associated with the basal

ganglia and declarative learning associated with the hippocampus (e.g., Knowlton, Mangels & Squire, 1996; Poldrack, Probkharan, Seger & Gabriela, in press). Declarative learning results in the acquisition of various facts such as the fact that  $3+4 = 7$ . Procedural learning results in the acquisition of production rules that retrieve this information to solve particular problems. Thus, a student might be in the midst of solving the following multi-column addition problem:

$$\begin{array}{r} 336 \\ +848 \\ \hline 4 \end{array}$$

If the student is in the midst of working on the multi-column addition problem above, the next production to apply might be:

IF the goal is to add  $n1$  and  $n2$  in a column  
and  $n1 + n2 = n3$   
THEN set as a subgoal to write  $n3$  in that column.

This production would retrieve the sum of 3 and 4 from declarative memory and embellish the goal with the information that 7 is the number that should be written out. Then other productions would apply that might deal with things like processing the carry into the column. The basic premise of the ACT theory is

that cognition unfolds as a sequence of such production-rule firings. Furthermore, ACT implied that learning involved the acquisition of such production rules.

In the 1980s and 1990s we engaged in a process of analyzing the production rules that were necessary to achieve competence in a number of domains of mathematics and computer programming. The following are typical of the production rules that we proposed:

#### Lisp

IF the goal is to get the nth element of the list  
THEN code "car"  
    and set as a subgoal to get the  
    n-1st tail of the list

#### Geometry

IF the goal is to prove two triangles congruent  
THEN set as subgoals to prove  
    corresponding parts congruent

#### Algebra

IF the goal is to solve an equation in x  
THEN set as subgoals to graph the right  
    and left sides of the equation  
    and find the intersection point(s)

Given the assumption that learning in these domains involves the acquisition of such production rules, it follows that we should try to diagnose whether students have acquired such production rules and provide instruction to remediate any difficulties they have with specific rules. This led to the design of cognitive tutors that ran production-rule models in parallel with the students and attempted to interpret student behavior in terms of the rules in the models. The tutors use computer interfaces in which students can solve problems – for instance, a structured editor for programming or a graphical proof-planning interface for geometry. A process called **model tracing** tries to find some sequence of productions that produces the behavior exhibited by the student. This interpretation of the behavior controls the tutorial interactions in 3 ways:

(1) If students ask for help it is possible to give help appropriate to where they are in the problem-solving process, thereby individualizing instruction.

(2) If students appear to be progressing correctly the student model updates its confidence that they know the rules and therefore promotes

them through the curriculum. Thus, there are cognitively-based mastery criteria for promotion through the curriculum.

(3) If students make what appears to be an identifiable error it is possible to intervene and provide appropriate instructions.

For purposes of later discussion there are two features to emphasize about this approach. First, the tutor only has access to a narrow “window” of student behavior: mouse clicks, key presses, and their directions from which to make its inference about the student’s thinking processes. Second, the typical time span of these steps is on the order of 10 seconds or more. We will describe later in this paper what happens when we consider other behavioral indicants and cognition at a finer grain size.

## **Validity of the Approach**

There are two ways to assess the validity of this approach and the assumptions on which it is based. The first is by assessing its educational outcomes and the second is by looking in detail at the behavior of students working with the tutor.

## **1. Achievement Gains**

There are a number of assessments (Anderson, Corbett, Koedinger, & Pelletier, 1995; Koedinger, Anderson, Hadley, & Mark, 1997) which converge on the conclusion that students using these tutors perform about a letter grade (approximately one standard deviation) better than students who do not. The approach also appears to produce more motivated students and considerable teacher acceptance. It is harder to know how to measure motivation and acceptance but one thing we can point to is the fact that the tutors are currently used in about 100 schools around the country and teach about 15,000 students. This practical success in acceptance reflects a lot more than the achievement gains associated with the tutor. An analysis of just what is behind the practical success of the tutors remains to be done.

From the perspective of cognitive psychology, the achievement gain is the most interesting aspect of the evaluation. Figure 1 shows the results of a typical evaluation. This was an evaluation of the PAT Algebra 1 tutor that targets teaching problem solving skills and use of mathematical relationships. As is apparent it was having large positive effects (compared to the control) on these

skills and smaller but still positive effects on traditional measures of algebraic performance which it was not targeting. For the targeted skills the effect size was on the order of one standard deviation.

Insert Figure 1 About Here

For two reasons, it is quite problematical to know how to assess a one standard deviation achievement gain. First, it is hard to say how big an effect this is. One standard deviation achievement gain means that the distribution of student performance with the tutor is shifted up about one standard deviation from the distribution of students without the tutor. Since standard deviation is measured with respect to variance in the population this says as much about the population variability as the effect size. Second, it is hard to know how large an effect could be achieved. Bloom (1984) is famous for claiming an effect size of two standard deviations for human tutoring. However, other assessments have produced smaller effects (Cohen, Kulik, & Kulik, 1982).

Despite the uncertainty about how big an effect is possible, it is generally conceded within our group that our tutors are not having as positive an

instructional effect as possible. That is, students should be able to learn more per unit time than they are with the tutors. One sign of this is the low absolute levels of performance in Figure 1. There clearly is a lot of room for improvement. These data come from regular urban mathematics classes and are undoubtedly impacted by difficulties of lack of preparation, non-attendance, etc. Still, we know that there are times when students are not getting all they should from their instruction. For instance, there are numerous examples of profitless interaction where the student has a misunderstanding that the tutor cannot address. The fact that, despite these weaknesses, they are doing better than control classrooms might be seen more as an indictment of the control classrooms than as praise for the tutoring manipulation.

## **2. Analysis of Tutor Interactions**

The key assumption in these tutors has been that competence in a domain decomposes into a set of components and that the overall learning of the skill is just the learning of the individual components. This is a view of competence that many educators do not share, and the opposite opinion informs many educational programs such as those produced by radical constructivists in mathematical education (for a discussion see Anderson, Reder, & Simon, 1998). The hard-line

constructivist view (Lesh & Lamon, 1992; Shepard, 1991) is that it is not possible to break down a competence into its components, and any attempt to do so will lead to failed education. The success of our tutors clearly contradicts this.

Insert Figure 2 About Here

In addition to their overall success, the tutors afford more direct evidence on the issue of the decomposition of knowledge. In tracking knowledge, the cognitive tutors keep track of how well students are doing on various components. Figure 2 shows some data making this point from Corbett, Anderson, and O'Brien (1995). The data come from the LISP tutor. Students are asked to write a number of LISP functions and the figure shows their error rate at each point in each function across the early material. Students show a lot of jumps and drops in their error rates at different points. As the figure shows, our model successfully predicts these jumps and drops. That model assumes that each production rule has its own learning curve. Figure 3 averages together those points where the same production rule is applying. As Figure 3 illustrates, there is smooth learning over these points. Thus, the reason for the rises and falls in Figure 2 is the fact that new production rules are being introduced at different points in the curriculum. The

success in accounting for complex behavioral profiles like Figure 2 is for us the most compelling support for the decomposition of knowledge. However, it needs to be noted that this success depends on a very careful task analysis. The original production rule set in the LISP tutor did not yield such regular learning data. The final set resulted from an iterative process of refinement. Indeed, we view the ability to come up with such systematic learning curves as a measure of whether the right production rules have been identified.

Insert Figure 3 About Here

### **The Grain Size of Cognition**

There is growing evidence that despite their successes, the cognitive tutors are modeling cognition at too large of a grain size and consequently failing to capture some of the regularities in learning. While results like Figure 2 are compelling evidence for decomposition, there is reason to believe that these learning curves actually reflect the aggregate results of a variety of simpler cognitive steps. Research in the ACT-R laboratory has supported the decomposition of cognition into sub-second units. In one study of a simulated air-traffic control task Lee and

Anderson (submitted) found that the overall task decomposed into unit tasks of the multi-second level typically modeled in the past generation of tutors. However, more careful analysis indicated that each of these unit tasks decomposed into simple actions of moving visual attention and issuing individual keystrokes.

Movement in the direction of a finer temporal resolution is not something that is unique to the ACT research group. Indeed, reflecting an emerging consensus in cognitive psychology about the grain size of cognition, the architectures for human cognition have all moved to a small cycle size. In a symposium on production system architectures at the 1995 Cognitive Science meeting (Anderson, John, Just, Carpenter, Kieras, & Meyer, 1995), it was noted that the four production systems (ACT, Soar, EPIC, and CAPS) had all converged to a minimum cycle time of only 50 ms. This reflects the efforts of these cognitive architectures to accommodate the detailed data about human cognition.

The strongest regularities in human cognition have come from studies of learning that look at small units of knowledge and studies of performance that look at brief tasks. These tasks are typically on the order of one second in length, and

subtracting out encoding and motor time reveals that the cognitive component is often only a few tenths of a second.

Many educators decry the emphasis in the classroom on problems that span only a few minutes. They argue that it leads the child to believe that all problems only require this amount of effort. Still, the contrast is at least 3 orders of magnitude between the time scale of these classroom tasks and the few hundred milliseconds that cognitive psychology has identified as the place where the real regularities appear in cognition. This contrast has led to an unfortunate disconnect between much of cognitive psychology and much of education.

Many educators do not believe that anything at the sub-second time scale could possibly be relevant to understanding the tasks of significance they study. As we already noted, some hold the belief that task decomposition is just not possible. We think these beliefs are wrong and that we have the evidence to show that such task decomposition is indeed possible. Others, however, just believe as a practical matter it will not be possible to do the work to develop such decompositions for the learning phenomena of interest. The cognitive tutoring work involves task decompositions probably two orders of magnitude above the level identified in

modern cognitive psychology. It is a legitimate question to ask whether it is a bridge too far to go from the low-level cognitive tasks popular in laboratory psychology to educational applications.

For their part, many cognitive psychologists distrust the research that involves complex tasks spanning time scales of many tens of seconds. They hold the view that such complex tasks are inherently too messy to be given systematic analysis. Again we think the cognitive tutoring work has proven this view to be false. On the other hand, there is a weaker and more reasonable bias. This is that the 1-second laboratory task offers the necessary control and ability to isolate phenomena of interest. A similar concern over experimental control and the isolation of specific phenomena makes researchers in cognitive neuroscience reluctant to image complex tasks because they fear that too many neural areas will be involved and it will be impossible to pull them apart. While these are legitimate concerns, their consequence is that there has been a failure to make the connection between basic cognitive psychology and education.

Perhaps the correct solution is to introduce another island along the path from neuroscience to education. In this conception, there would be three bridges

altogether. The bridge from neuroscience to basic cognitive psychology would be one that aggregated neural detail into units of psychological significance. The bridge from these basic cognitive processes to something like the unit tasks in our cognitive tutors would reflect an aggregation of these units into units of educational significance. The bridge from this unit-task level to education would show how to construct practical applications.

While this 3-bridge proposal has a lot of plausibility, this paper is devoted to exploring the idea that the basic sub-second cognitive level can inform the multi-minute level of education. We will show that instructional opportunities become available by attending to what is happening at the sub-second level.

## **The ACT-R Theory**

We will close out this section by describing the ACT-R theory, which is a cognitive architecture that embodies this sub-second level of analysis. Figure 4 illustrates the control structure in ACT-R. Cognition is controlled by the current goal, and a production is selected which matches the current goal. This production can cause both actions to be taken in the real world and retrieval requests to be

made of declarative memory. The actions in the external world can include shifts of attention, which in effect also retrieve information from the external world. This retrieved information results in changes in the goal and the cycle can progress again. This cycle of production firing, retrieval, and goal modification takes on the order of a few hundred milliseconds or less. Thus, it reflects the loop of cognition that has become the focus of analysis in basic cognitive psychology.

Insert Figure 4 About Here

The inner loop in Figure 4 reflects a slower learning loop by which knowledge structures can be modified. New chunks are added to declarative memory when goals are achieved. For instance, if a child sets the goal to add  $4+3$ , counts up, and finds 7 as the answer<sup>2</sup>, this goal is popped and stored as a chunk that can be retrieved with the answer. Chunks are also formed by encoding information from the environment. New production rules can be formed by compiling representations of past solutions in declarative memory. Finally, production rules can call for pops and pushes of goals that change the goal state. Pushes store an intention and focus on a more immediate goal. Pops retrieve an old intention and supplant the current goal with it.

The cognitive architecture in Figure 4 reflects an abstraction from underlying neural processes, as indicated by the labels bearing probable brain correlates. The goal state is really the current active cortical state. There is evidence (Wise, Murray & Gerfen, 1996) that the basal ganglia receive activation from the full cortex and recognize patterns and send these recognized patterns to the frontal cortex, which selects an appropriate action. Thus, production memory is really implemented in basal ganglia and frontal cortex. There is a great deal of evidence pointing to the hippocampus as playing a major role in the creation of declarative memories and the frontal cortex as responsible for the maintenance of intentions, which is abstracted in ACT-R as the goal stack.

When we talk about ACT-R in terms of chunks and productions we are treating cognition as a symbolic system, which is a useful abstraction. However, productions and chunks vary in their availability which reflects the underlying more continuous neural computation. There is a subsymbolic level of ACT-R that models cognition at this level. Much of the important learning in ACT-R is at this subsymbolic level and involves making chunks and productions more available with practice and success. We do not model actual neural learning processes, but

rather model their effect by a set of equations that characterize these processes. In this way, ACT-R is an attempt to abstract away from the neural detail to what is relevant to human cognition. These subsymbolic learning processes are very important to learning. Anderson and Schunn (in press) have written about their implications for educational practice.

When one models cognition at a fine grain size it becomes important to consider the relationship of cognition to perception and action. Therefore Byrne (Byrne & Anderson, 1998) has created an extension of ACT-R called ACT-R/PM, which is represented in Figure 5. All of ACT-R from Figure 4 is embedded in the cognitive layer of Figure 5. In the perceptual layer are a number of independent modules that control hand movement, speech, movement of visual attention, and movement of auditory attention. Each of these peripheral modules is capable of running in parallel with cognition and with each other. However, each of these modules is serial within itself, only doing one thing at a time even as ACT-R can only fire one production at a time. This leads to a complex set of predictions about exactly what can be done within a specified period of time, and these predictions seem largely to be confirmed (Byrne & Anderson, submitted).

Insert Figure 5 About Here

Initial control of each of these modules in Figure 5 seems to begin in the frontal cortex but each action module and the perceptual module involves a great deal of other neural structures. It is sobering to realize that much more cortex is given over to perception and action than things we would associate with ACT-R or other cognitive architectures. This should make us aware of how important it is to consider human perception and action in a complete account of human cognition.

### **High-Density Sensing**

We have argued above that significant acts of cognition are happening at the sub-second level and that at least some of them have behavioral indicants. Thus, it is possible to increase one's sensitivity to students by monitoring their behavior at a high temporal density and using this data to make inferences about student cognition. One advantage of human tutors over computer tutors is that humans have such fine-grain access to their students' behaviors. This allows for far greater sensitivity and adaptability in a tutorial interaction. A human tutor can see frustration on the pupil's face, hear uncertainty in an utterance, and is aware of

how long the student is taking to solve a problem. These are examples of types of input that are typically not used by computer tutors. VanLehn (1988) referred to the amount and quality of the input available to a tutor as an issue of "bandwidth." He identified three bandwidth categories which constitute three different levels of information regarding problem solving processes. In increasing order of information detail they are: final states, intermediate states, and mental states. A "mental states" level of bandwidth is necessary to support the model tracing approach used in the ACT tutors, and this can be done with keystroke and mouse click data. However, technological advancements and recent changes in the grain-size at which cognition is analyzed in the ACT-R theory have led us to speculate that additional instructional opportunities are afforded by increasing the bandwidth even further.

There are obvious indicants of confusion or understanding such as facial expression and speech. One might think the student could type the same information that they speak but indeed students find it much easier to speak their minds than to type their minds. A frequent occurrence in the tutor classrooms is that students will point to something on the screen and ask somebody "What does

this mean?" (Where they could ask the same questions of the tutor with clicks and keypresses.)

Of most importance to the main point of this paper, humans also have an ability to monitor what students are attending to visually. For instance, in a study of human accuracy in gaze estimation, Gale (1998) found that humans could estimate where someone was looking with a root mean square error of about 2 to 4 degrees.

When construction of the cognitive tutors started in the 1980s it was beyond the realm of possibility to think about using information at the grain size of speech or eye movement. The computers were hard pressed to keep up with an interpretation of students' keypresses and mouse clicks in real time. Things have dramatically changed on this score. Speech recognition software now comes as a standard component with many computer systems. Mostow (1998) has shown that using speech recognition software, which is only modestly more tuned than the standard commercial software, one can monitor and instruct students' reading. His demonstration is important because it shows how one can use the constraints of the instructional situation to facilitate speech recognition. It is also now possible to attach a camera to a computer, monitor the student's face, recognize

expressions and emotions (Cohn, et al., in press), and to track where the eyes are gazing. In our own lab we have shown that it is possible to compute interpretations of the student's eye movements in real time (Salvucci, 1999). Thus, it is now possible to think of the computer tutor as having as rich a perceptual access to the student as does a human tutor.

There are three significant issues that such access raises. The first is more apparent than real but needs to be addressed: privacy and intrusiveness. The second is the core of the instructional issue. This is what instructional opportunities are created by such high-density access. The third is the cognitive and technical issue, which is how to place a cognitive interpretation on the information that one gets from such high-density sensing. We will address each of these issues in turn.

## **Privacy and Intrusiveness**

Some people set up Web pages where they provide camera access to their private lives. However, most of us (and most students) would find abhorrent the prospect of the world having such access to our lives. On the other hand,

virtually no one finds abhorrent the fact that a computer is tracking how they are moving the mouse. This reflects the fact that we want the computer to be sensitive to what we are doing but we do not want it to be a means of broadcasting (or recording for the world) what we are doing.

The situation with respect to the proposed high density sensing is somewhere in between the two examples above. Just as current speech recognition software does not keep a permanent record of all the speech (not just out of concern for privacy, but because it would be too much information), any sensing technology would not store a record of the entire sensory signal it was getting from the student. As with speech or mouse movements, it would extract an interpretation of the student from the raw signal. Note that this is equally true of human tutors, who do not remember exactly what they heard and saw. On the other hand, in the tutoring situation there is a natural desire to maintain a permanent record of that interpretation both for purposes of informing further interactions with the student (this is what individualizes the tutor dialog to the student) and for purposes of later assessment. Despite this, it is probably the case that the actual interpretation of the student will be no different than the interpretations currently stored of the students in our tutors. There will not be more detailed records

kept—just more accurate. Typically, our tutors only keep summary estimates of how well students are doing on particular cognitive objectives (for future diagnosis) and which problems students have done (for teacher information and to inform future problem selection). Except in experimental situations, we do not keep even a high-level record of the key and mouse interactions. For example, we have never recorded in our tutors mouse trajectories over the screen. Therefore, despite some people's initial reactions to the contrary, the introduction of high density sensing does not change the recording of the student beyond what one would find in a teacher's grade-book. The only difference is the degree to which the record reflects accurate cognitive diagnosis.

Intrusiveness is a somewhat different issue than privacy. Interacting with a high-density tutor will be very much like interacting with a human tutor, where every gesture will be interpreted. This human-like quality might seem a clear benefit, but there is an intrusive nature to such interactions that might not always be desired. Students sometimes like the ability to do their work out of the public eye. A frequent positive comment about our computer tutors from students is that they feel more at ease with them than human tutors because the computer tutors appear less judgmental. Because of its potential intrusiveness, it is not

obvious that high-density sensing would always be a win and one might well want to have the opportunity to turn the high-density sensing off or, indeed, give the student that option. Giving the option to turn the high-density sensing off is really no different than the options that now exist to use or not use the tutor. We will just be providing the student and teacher with another option in their learning interactions.

## **Instructional Opportunities**

While speech and face recognition offer potentially powerful ways to give computer tutors better access to the student's mental state, our focus in the rest of the paper will be on eye movements as a window into the student's mind. We have done research on the instructional opportunities afforded by eye movement information in the context of the PAT algebra tutor. The ninth grade algebra tutor, PAT, is the most widely used of the cognitive tutors. (Figure 1 shows the results from an earlier version). Therefore, it seemed wise to focus on the opportunities for eye movements to inform instruction here. The PAT tutor is concerned with teaching students to relate graphical, symbolic, verbal, and tabular representations of mathematical functions. The graphing features are only introduced after a

couple of lesson units and for technical reasons we chose to study the version of the tutor from the early lessons without the graphing interface. To help eye movement resolution we somewhat altered the exact size and arrangement of material on the screen but otherwise deployed a system which faithfully reproduces the instructional interactions in the actual classroom.

Figure 6a shows a screen display as it appears at the beginning of a problem. The student's task is to fill in the column labels and units, enter a variable and an expression written in terms of that variable, and then answer whatever result-unknown and start-unknown questions are present. Figure 6b displays the completed problem. The key aspect of the problem is the expression  $12 + 45x$ , which the student has filled in. The real goal of the lesson is to teach the student how to create such expressions and use them to solve problems.

Insert Figure 6 About Here

There are a total of 18 problems, all similar in form to this one. Two of the problems are used strictly for introducing students to the task, and the other 16 are completed while the student is calibrated on the eye tracker. Each participant

completed these problems at a rate of four per day, for four days in a row. The participants were students in pre-algebra and beginning algebra classes, as are the students who use the PAT tutor in actual classrooms.

The eye tracker used in this study is a head-mounted unit designed by ISCAN, Inc. It is lightweight and rests on the student's head like a visor. Low-level infrared illumination provides a corneal reflection that is used with the pupil center to provide point of regard (POR) estimates.

Our research with the system is concerned with collecting descriptive statistics of eye movements and characterizing learning trends. Having access to such visual attention data allows us to infer a great deal about a student's problem solving processes. In cognitive psychology, there is nothing new about using eye movements to reveal the microstructure of a cognitive process. It is new, however, to investigate ways that we can leverage off this sort of data in a real-time tutoring situation. One of the primary motivations for this project is to explore the kinds of instructional opportunities brought about by increasing the bandwidth of information available to a computer tutor. We would also like to develop an ACT-R/PM model that is capable of interacting with the tutorial

interface in the way students are. However, for current purposes we will just report some of the types of instructional opportunities that we have found. These opportunities reflect different ways that eye movements enhance our ability to diagnose the student's cognitive state. We will consider six examples of such instructional opportunities.

In the sections that follow, we will provide examples of typical eye movements that indicate an instructional opportunity. These eye movements are represented as a series of "blobs" on the screen that show where the student's eye was fixated during the episode. These fixation blobs move from light to dark with the passage of time. We will also provide statistics to indicate how representative these individual examples are. Each fixation typically occupies a few hundred msec.

### **1. Student has Shifted Attention to a Different Part of the Problem without Informing the Tutor.**

The tutor problems are typical of many significant mathematical problems in that they have a number of components and it becomes an issue to make sure that the tutor knows what part of the problem the student is currently thinking about.

The student can be working on one part of the problem while the tutor thinks the student is working on another part of the problem. A classic example of this that occurs in the PAT tutor is that the student selects one of the columns but gives the answer for the other column. For instance, a number of our students clicked the first column in which one enters givens but then went ahead and calculated the result that goes into the second column. Not surprisingly students display very different eye movements when they are entering the given versus calculating the results. Figures 7a and b display a typical contrast. Figure 7a shows a successful episode in which the student has fixated the 5 hours in the first question and entered the 5 in the column. The few fixation blobs indicates how brief and direct the eye movements are. In contrast, in Figure 7b the student chose the cell to enter the given but has gotten ahead of himself and calculated the altitude rather than simply entering the time. Note all the eye fixations over the problem statement — an indicator that the student is calculating the result and not entering the given.

Insert Figure 7 About Here

The error of putting the solution in the given cell may not seem like a particularly interesting error. That is exactly the problem. From an instructional standpoint, it is not an interesting error. This is not an impasse arising out of some fundamental conceptual error that the student could learn about and overcome by working through the error. There is little, if any, additional learning gain that arises out of a student's having committed this error. Not only is there little to be gained, but there sometimes is considerable confusion and frustration that arises out of having committed this error. Students do not always see the bug message that appears ("You typed that in the wrong column."), and when they do not see that message, and they are pretty confident they did the computation correctly, it is hard for them to understand why the tutor considers it wrong. Some students even re-enter the solution in the wrong cell, to see if the tutor will accept it *this* time. Given the lack of instructional utility and the confusion it sometimes creates, it is an error to be avoided if possible. One way of avoiding this error is to use the student's eye movements to detect the cognitive attention switch and intervene before the error is committed. Ignoring for now the issue of what the intervention should be, we will first explore the possibility of detecting these cognitive attention shifts on the fly.

The eye movement data used in this analysis were those collected during 1<sup>st</sup> attempts at cell Q1-Left (the cell on the left side of the row for responses related to Question 1). Because the students completed 16 problems on the eye tracker, and there is one 1<sup>st</sup> attempt at each cell per problem, this results in 16 part-tasks from each student for this analysis.<sup>3</sup> Eye movement data were extracted from the beginning of each part-task to just before the first character of a response was typed. Examining responses from each student individually (on 1<sup>st</sup> attempts in Q1-Left only), it was found that six students committed this error three or more times. It is the data from these six students that are used to try to predict a cognitive attention shift based on eye movement data.

We started with a shotgun approach to predict when such a shift of attention would occur. The predictor variables are fixation counts and gaze time in every point-of-regard (POR) region, as well as latency from the beginning of the part-task to the first keypress. Gaze time in a POR region is computed as the sum of the fixation times for all of the fixations in that region. If we include fixations offscreen, fixations to the keyboard, and fixations to MON (“Middle Of Nowhere” – fixations on the screen that are not in any of the defined regions), there are 23 different POR locations. With two variables (fixation count and gaze)

for each region, plus the latency variable from the beginning of the part-task to the 1<sup>st</sup> keypress, we have a total of 47 predictor variables available.

The two variables with strongest correlation with probability of an attention shift were number of fixations on the problem statement and amount of time fixating the problem statement. After either of these were added to the predictive equation no other variable had much of a correlation. Figure 8 shows the probability of an attention shift as a function of number of fixations of the problem statement. It is apparent that once the number of fixations equals or exceeds 15, it is almost certain that there will be an attentional switch.

Insert Figure 8 About Here

One needs to be fairly conservative in predicting an impending attention shift error, and this is exactly what one would want in a real-time tutoring situation. Imagine how frustrating it would be to find out the tutor thinks you are about to make an error, when in fact you are quite aware that you were *not* about to make that error. Ideally, the tutor would only intervene when it is highly confident that

the student is headed into a mistake, and it can help the student avoid the additional confusion and lost time.

Given these promising results, we are confident that it is possible to identify cognitive attention shifts of this sort and enable the tutor to be more closely aligned with the student. It is another question what to do in such a situation. We used the generic term “intervention” several times in the preceding discussion. What might this intervention be? One possibility is to be reactive and simply tell the student after the fact why the answer was an error. With respect to this latter possibility we should note that, as it currently exists, the tutor will give the student such an error message should they type the value that belongs in a different column. However, eye movements would allow the tutor to be more confident in that diagnosis and identify cases where students try to perform the calculation and make a mental arithmetic error instead. We will give an example of such a disambiguation in (4) below. Another proposal is to be proactive and warn the student with a (perhaps spoken) message “You should be working on the given.” A final possibility, and one that is especially intriguing, is to design the tutor to modify itself to accommodate the assessment that a cognitive attention shift has taken place. For instance, if the model is predicting an attention shift

error for the current part-task, the tutor could simply change the cell selection so that the solution cell (Q1-Right) is selected. When the student starts typing, the solution would be entered in the correct cell, thanks to the eye movement data.

## **2. Disambiguation of solution method.**

Students do not all solve these problems in the same way. With respect to the algebra problems we have been working with, differences in solution paths are apparent by examining how students calculated the solution. An interesting contrast is between students who use the algebra expression to calculate an answer and students who go back to the verbal problem and reason from that. Koedinger and colleagues (Koedinger & Anderson, 1998; Koedinger & Tabachneck, 1995) have reported that students sometimes find it easier to reason about word problems than to perform the analogous symbolic manipulations, and Koedinger and MacLaren (1997) have been developing a model in which there are both algebraic and verbal methods for solving problems. Our eye movement research clearly validates that distinction and allows us to tell which method a particular student used to produce an answer. Figure 9a shows a subject using the expression and Figure 9b shows a subject using the problem statement. In the case where the expression is used the student shows repeated fixations of the

expression. In the case where it is not used the student never fixates the expression but rather fixates the problem statement while reasoning verbally through the problem.

Insert Figure 9 About Here

One can wonder whether students are more accurate when they calculate the answer using the expression in the problem statement. To address this question, we looked at fixation frequencies in the problem statement and the expression while participants were working on their 1<sup>st</sup> attempt at calculating the answers. Each part-task was labeled as to the presence of fixations in the two regions of interest. If there were one or more fixations in the expression, but not in the problem statement, the part-task was labeled “expression.” If there were one or more fixations in the problem statement, but not in the expression, the part-task was labeled “problem statement.” Part-tasks were also labeled as “both” or “neither,” as appropriate. Table 1 shows the results in the form of the percentage of part-tasks that fell into each of these categories and the proportion of correct 1<sup>st</sup> attempts within each category.

Insert Table 1 About Here

At the aggregate level, these data indicate that students looked at the expression 54% of the time on their first attempt at solving Question 1, suggesting that the traditional model of this process is about half right. The accuracy data show an advantage for using the expression over using the problem statement. There is a main effect of fixation pattern ( $F(3, 142) = 3.68, p < .02$ ) on accuracy, and a post-hoc Scheffe's test indicates that those who attended to the expression (but not the PS) while solving the result-unknown responded more accurately than those who looked at just the problem statement ( $p < .02$ ). The other differences are not significant. There is also a main effect of fixation pattern on part-task completion time ( $F(3, 142) = 19.69, p < .001$ ), with attending to neither region significantly faster than all of the other fixation patterns and attending only to the expression faster than attending to just the problem statement or to both.

One goal of the curriculum is to help students understand the value of mathematical expressions. With respect to solving result-unknown questions, the expression is useful in that it provides external memory for the computations required to arrive at a solution, and our performance data do indeed show an

advantage for those part-tasks in which students looked at the expression. These strategies cannot be distinguished on the basis of answer alone, but they *can* be distinguished by a tutor that has access to a student's eye movements. While the details need to be worked out, the obvious instructional intervention would be to guide the student in the use of the expression.

### **3. Failure to Read Messages.**

One thing that became apparent with our very first pilot subject is that students often fail to read messages that appear on the screen. For instance, often a student will make an error, the tutor will correctly diagnose that error and present a bug message, but the student will not read the message. Figures 10a and b present an interesting example which are two parts of a student's reaction after making the classic error of entering a result in the given (same kind of error as in 1 above). The tutor presented the error message "You typed that in the wrong column." Figure 10a illustrates the student's eye movements for the first 18 seconds where the student failed to read this message and looked through the problem for a possible explanation of his error. Finally, in Figure 10b the student looks at the error message and quickly corrects his error.

Insert Figure 10 About Here

The even more extreme case is when a student never reads the bug feedback at all. We saw examples of this during data collection, and became interested in how often this actually happened. To investigate that we extracted data from only those part-tasks that immediately followed a part-task where an error occurred and led to the display of a bug message. So in all of the part-tasks used in this analysis, the student has just committed an error and there is a bug message on the screen. We also have used only the data from the beginning of each part-task to the first mouse click (a click clears the message window). Figure 11 shows the high number of part-tasks in which students did not look at the bug message at all. They failed to fixate the message window in 41 percent of the part-tasks in which a bug message was present.

Insert Figure 11 About Here

The instructional response to such failures to pay attention might seem obvious. Students could receive a spoken prompt to read the message or perhaps we should

flash the message to grab attention. There is a possible complication here, however, which is that the tutor messages are not always perceived as useful by the students. For some time now, there have been anecdotal observations of students who, with experience, seem to learn *not* to pay attention to these messages. It turns out there is little evidence for this in the eye tracking data. Table 2 lists the proportion of bug messages that are ignored (i.e., no fixations in the message window) across the four days eye movement data were collected. There is somewhat of an increase on the last day, but this hardly constitutes the sort of clear trend one would want to use as evidence that students are learning to ignore the bug messages.

Insert Table 2 About Here

Another way the eye movement data are useful in this context is to help in the evaluation of messages. It is interesting to know whether the help and feedback messages are having their intended effect. One might think that this issue can be investigated in the absence of eye movements. This is true, but in that case the analysis is done under the *assumption* that the student is reading the message when it is displayed. As is evident in Figure 11, above, this often is not the case.

Eye movement data allow the researcher to determine with certainty whether a student has read a message. If a student gets some bug feedback and then fails to correct the error, we would not want to blame the message if the subject did not read it. We should note that sometimes a subject makes an error, a message is presented, and the subject corrects the error without ever having read the message. In this case, we would not want to credit the message. Thus, eye movement data make it possible to identify which messages, when read, lead to improved performance.

#### **4. Disambiguation of an error.**

There are a number of situations where students make errors that are hard to interpret. One reason for this is that the same error could have multiple causes. While this is a frequent outcome in our tutors in general it does not happen in the portion of curriculum that we were studying. The other situation is where the student makes a miscalculation and produces an error that is bizarre. This does happen in our curriculum and eye movements offer an opportunity to disambiguate this class of errors. Figure 12 shows an example of this. The student has made the same mistake as in Figure 8a where the result is being

calculated in the cell where the student should put the given. However, the student has failed to correctly calculate the result, producing an error that would seem totally anomalous to our existing tutor. Note that the eye movements, however, reflect the tell-tale pattern of calculating a result. If the tutor were designed to do so, it could identify this pattern of fixations and tailor the feedback appropriately – perhaps noting in the message that the student’s solution is incorrect (and by the way, the solution is supposed to go in the *other* column!).

Insert Figure 12 About Here

**5. Student has failed to process some critical information for answering a question.**

Students are notorious for not reading problems carefully before answering them. A nice example of this appears in Figure 13. The subject has entered the date directly from Question 1, whereas the formula requires the subject to read the earlier problem statement and find the difference between 1985 and 1980. The student’s eyes never come close to the 1980 information. Again the instructional intervention seems obvious and is one that teachers are forever giving their

students “Read the problem statement”. An eye-tracking tutor can be more confident that the student has indeed failed to read the problem and be more certain that such a message is appropriate.

Insert Figure 13 About Here

## **6. Student is off task.**

There is another class of behavior which we have not seen in our laboratory work with the eye tracker but which we know happens with some frequency in the classroom: Students are just off task and are not looking at the screen. It is not clear that we want to take any actions to correct this behavior but it can inform tutor diagnosis. For instance, it would enable us to use latency of response to infer when a student is having real difficulties. In the current tutors, we do not know during a long-latency response whether the student is looking at the screen and is stuck, or perhaps has turned around and is talking to a neighbor or the teacher. If the student is busy scanning around the screen during this long latency then we can be fairly confident that they are indeed stuck or confused and can volunteer some help.

While we do not have instances in our laboratory work of students off task, we certainly have instances of students searching the screen for an answer. Figure 14 displays the bandwidth of information that would be available to an eye tracking tutor. More than 30 seconds have passed so far in this part-task, and the eye movements during this period reveal that the entire time is spent trying to arrive at a solution for this cell, with multiple saccades among the intercept, the slope, and Question 1. This is clear evidence that the student needs help, but the typical computer tutor would be aware only that the cell has been selected.

Insert Figure 14 About Here

## **Implications for Model Tracing**

Our goal in presenting these examples is to demonstrate that cognition at the grain size currently monitored in the tutors is supported by cognition at a much finer grain size and that we can gain instructional leverage by monitoring the student at this finer temporal resolution. These examples are also further support for the

general thesis that the complex cognition in the classroom can be decomposed into cognitive units of the sub-second level.

While the decomposition thesis is clearly supported, it does not follow that we have to model cognition at such a fine grain size to achieve our instructional leverage. Recall that our tutor models the problem solving in productions that span many seconds. All the examples that we described so far could be handled by recognizers that looked for a pattern of eye movement that was indicative of that solution method, that bug, or that failure in instruction processing. These recognizers could be attached to existing production rules. We would not really have to produce a process model that actually simulated cognition at such a fine grain size.

There are a lot of reasons to want to keep our instructional models at their current large grain size. Obviously, it is less work to model problem solving at this granularity. However, there are also serious problems of non-determinacy at the lower level of granularity. For instance, while there are patterns of eye movements that implement using the expression and others that implement using the problem statement, there is not a unique sequence of such eye movements.

Although ACT-R is a non-deterministic model that is perfectly capable of accounting for such variation and much of our research has dealt with distributions of behavior, computationally it can become intractable to trace the subject through a large non-deterministic space of cognitive states.

Salvucci (1999) has worked on developing an application of hidden Markov models for interpreting eye movements. Provided there are not huge numbers of states it achieves efficient recognition of the correct cognitive interpretation of eye movements. It is the sort of algorithm that would work well if we wanted to recognize the general patterns but which would become intolerably slow if we wanted to actually track the system through the exact ACT-R sequence of sub-second cognitive states.

## **Conclusions**

As a scientific statement, complex cognition can be decomposed into units of cognition of a few hundred milliseconds. We suspect there is no further decomposition of cognition beyond this point that does not involve going to brain models. The cognitive tutoring work has progressed at a much higher level,

analyzing cognition into unit tasks of often more than 10 seconds. As a practical matter, it may be that this is as fine a grain size as one can trace human cognition in an instructional environment. However, the most important properties of an architecture like ACT-R do not show through at this temporal grain size and really only show through at the sub-second level. We have shown that, even while one might not be able to model trace human cognition at such a fine grain size, behavior occurring at this grain size can inform instruction. It is an open issue just what the magnitude of the instructional leverage is at this grain-size, but it is possible that much of the advantage claimed for humans over computers might be due to the fact that they are sensitive to information at this grain size.

To return to the issue of bridges from the brain to instruction, it seems plausible that at least three bridges are required. The first would get from the brain to models of the simple steps of cognition which are primitive steps of cognition in ACT-R. The second would go from these components to performance of unit tasks. The third would compose these unit tasks into educational competences of true significance. ACT-R provides a system in which we can model these bridges. While ACT-R may inform the construction of the third bridge and facilitate

instruction, it does not seem profitable to cast our educational models at the level of detail of ACT-R models.

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## Figure Captions

Figure 1: Comparison of control and tutorial subjects on various tests of algebraic problem solving. Percent correct answers.

Figure 2: Actual predicted error rates across subjects at each goal in a set of required tutor exercises. From Corbett, Anderson and O'Brien (1995). Reprinted by permission.

Figure 3: Mean actual error rate and expected error rate for the ideal coding rules across successive rule applications. From Corbett and Anderson (1992).

Figure 4: A representation of the memories in ACT-R and the flow of information among them.

Figure 5: Overview of ACT-R/PM architecture.

Figure 6: The tutor screen at the beginning of the problem (a) and at the end of the problem (b).

Figure 7: Eye movements when a student does not make an attention shift (a) and when a student has (b).

Figure 8: Probability of attention shift as a function of number of fixations on the problem statement.

Figure 9: Eye movements when a student calculates the answer using the expression (a) and when a student calculates the answer using the problem statement (b).

Figure 10: Students eye movements in the first 9 seconds before reading an error message (a) and in the  $n$  seconds after (b).

Figure 11: Frequency of fixation in the window that contains the error message after an error.

Figure 12: Eye movements when the student has both miscalculated a result and placed it in the wrong column.

Figure 13: Eye movements when a student has failed to subtract the date, 1980, in the problem statement from the date, 1985, in the question.

Figure 14: Eye movements taking place when a student is having difficulty answering the question.

## Footnotes

1

This research is supported by grant CDA-9720359 from the National Science Foundation and by funding from the Air Force PALACE Knight program to Kevin Gluck.

2

Of course, only after repeated practice will this fact be strong enough to be successfully retrieved.

3

Except for two students who have missing data. One of them has 12 part-tasks total (lost an entire day of data), and the other has 15 part-tasks (lost one problem's worth of data). In both of these cases, the data were lost because the students managed to crash the tutor mid-task, before the data were written out.

Table 1. Distribution of fixation patterns during Q1-Right 1<sup>st</sup> attempts

Fixation Pattern	% of Part-Tas	Prop. Correc	Time (sec)
Expression (but not PS)	30.1	.88	13.2
PS (but not Expression)	12.6	.50	21.2
Both	23.8	.76	21.1
Neither	33.6	.71	7.7

Table 2. Proportion of Bug Messages Ignored Each Day

Day	Proportion Ignored
2	.41
3	.37
4	.40
5	.48

Figure 1

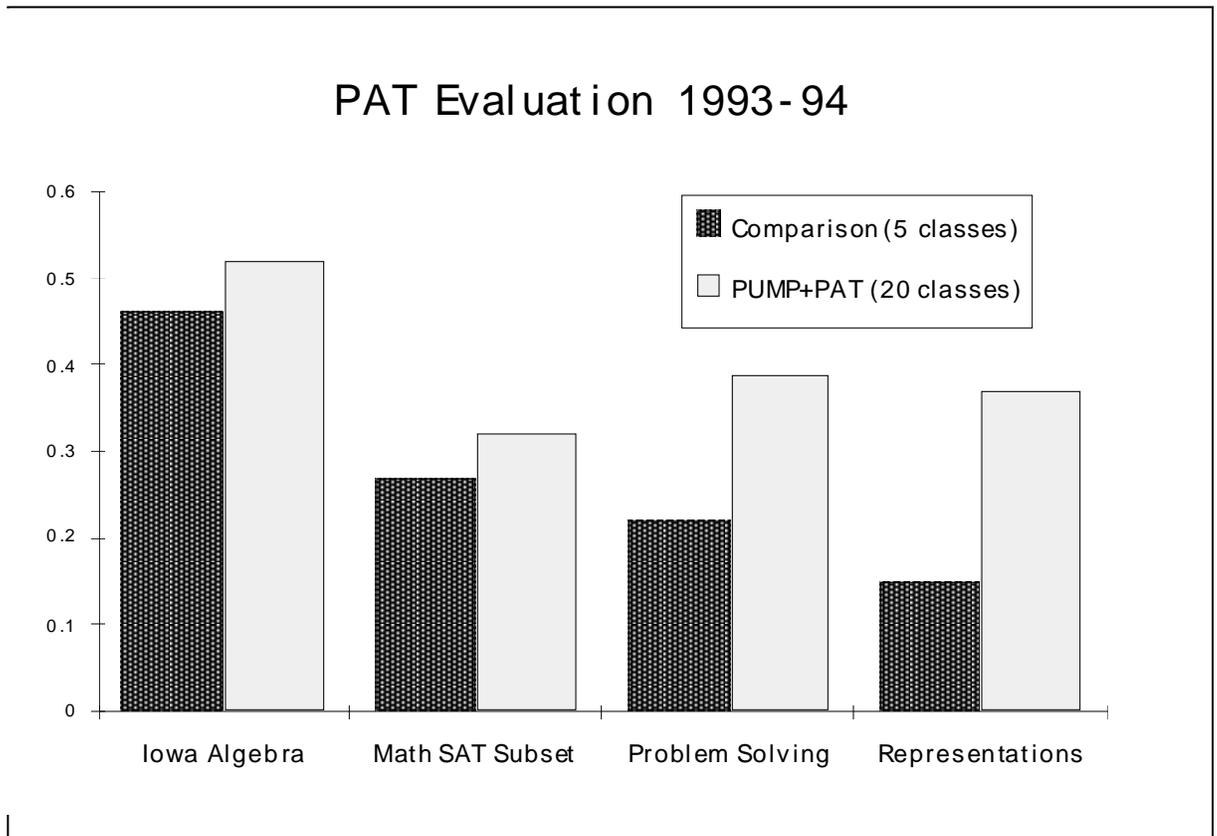


FIGURE 2

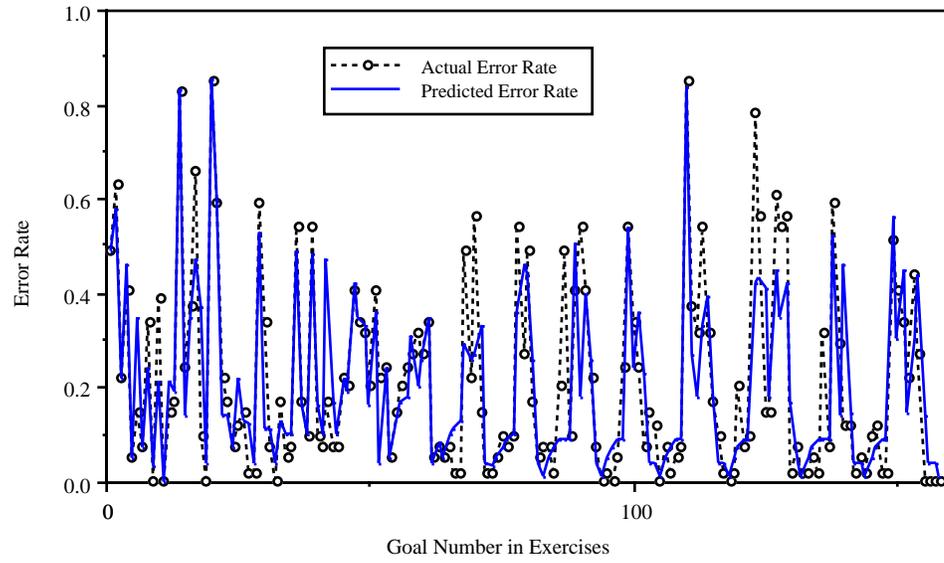


FIGURE 3

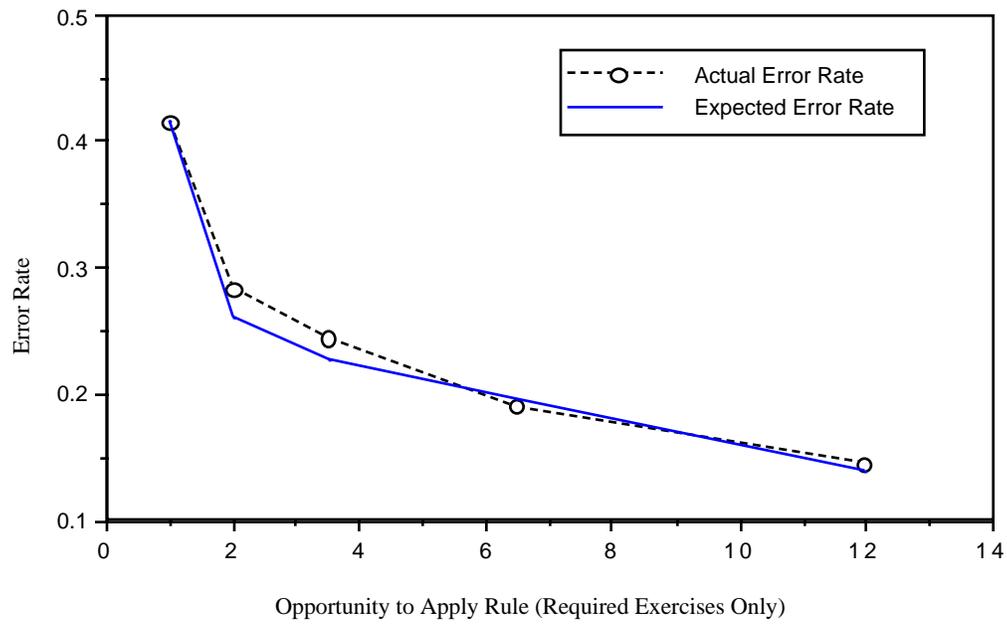


Figure 4

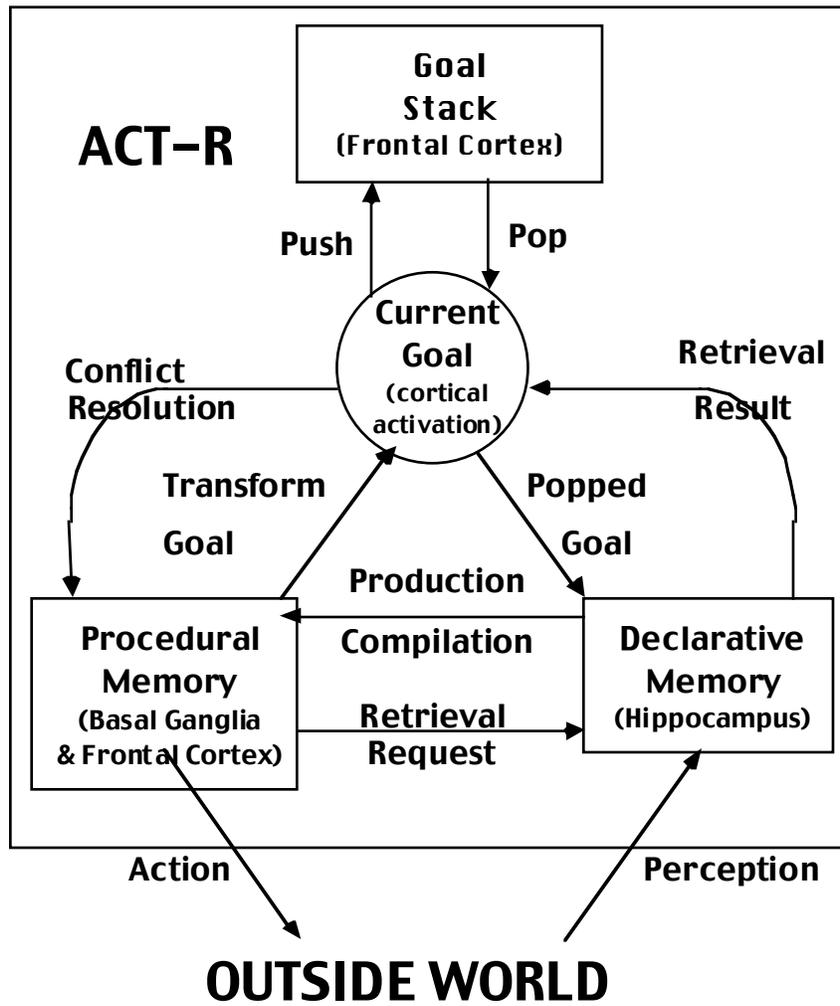


Figure 5

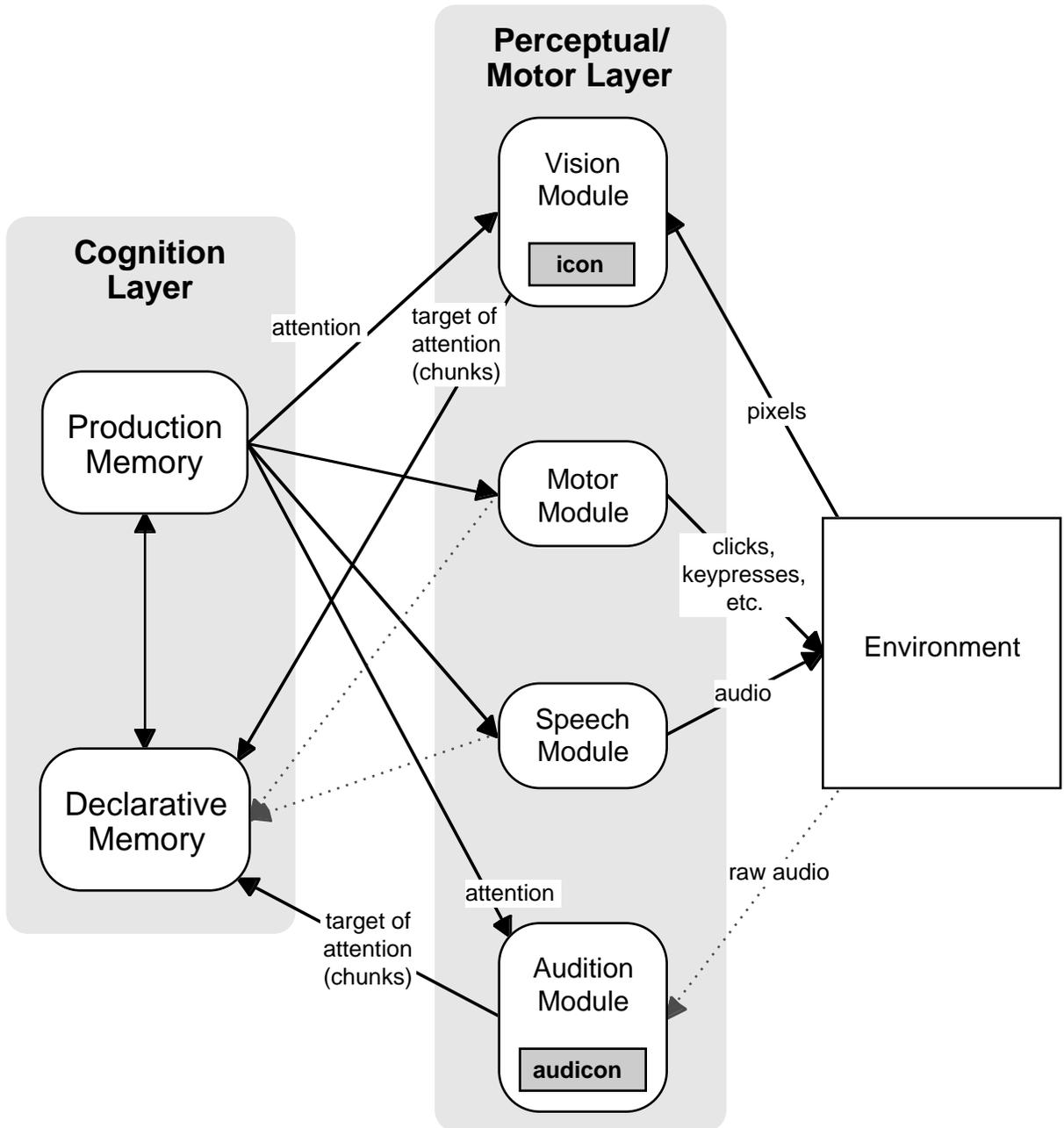


Figure 6a.

<p>Concert tickets cost 45 dollars a piece. A friend offers to stand in line to buy a number of tickets, if you will pay him a fee of 12 dollars to do so.</p>	<table border="1"><tr><td></td><td><input type="checkbox"/></td><td><input type="checkbox"/></td></tr><tr><td>Unit</td><td></td><td></td></tr><tr><td>Formula</td><td></td><td></td></tr><tr><td>1</td><td></td><td></td></tr><tr><td>2</td><td></td><td></td></tr></table>		<input type="checkbox"/>	<input type="checkbox"/>	Unit			Formula			1			2		
		<input type="checkbox"/>	<input type="checkbox"/>													
Unit																
Formula																
1																
2																
<p>Under this arrangement, how much would 5 tickets cost?</p> <p>What would be the total cost of 8 tickets?</p>																

Help

Done

<p>For the formula, define a variable for the number of tickets, and use this variable to write a rule for the cost.</p>
--

Figure 6b.

<p>Concert tickets cost 45 dollars a piece. A friend offers to stand in line to buy a number of tickets, if you will pay him a fee of 12 dollars to do so.</p>		<p># tickets    cost</p>	
Unit		tickets	dollars
Formula		x	$12+45x$
1		5	237
2		8	372

Under this arrangement, how much would 5 tickets cost?

What would be the total cost of 8 tickets?

Help      Done

For the formula, define a variable for the number of tickets, and use this variable to write a rule for the cost.

<p>You are driving at 10 miles per hour towards New York in the first gas car - talk about taking your time! Currently, you are a distance of 500 miles away.</p>	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 10%;"></th> <th style="width: 40%; text-align: center;">time</th> <th style="width: 50%; text-align: center;">distance</th> </tr> </thead> <tbody> <tr> <td style="text-align: right;">Unit</td> <td style="text-align: center;">hours</td> <td style="text-align: center;">miles</td> </tr> <tr> <td style="text-align: right;">1</td> <td style="text-align: center;">5</td> <td style="text-align: center;"><input type="text"/></td> </tr> <tr> <td style="text-align: right;">2</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> <tr> <td style="text-align: right;">Formula</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> </tbody> </table>		time	distance	Unit	hours	miles	1	5	<input type="text"/>	2	<input type="text"/>	<input type="text"/>	Formula	<input type="text"/>	<input type="text"/>
	time	distance														
Unit	hours	miles														
1	5	<input type="text"/>														
2	<input type="text"/>	<input type="text"/>														
Formula	<input type="text"/>	<input type="text"/>														
<p>What would be your distance from New York after 5 hours?</p> <p>After 12 hours had passed, how far from New York would you be?</p>																
<div style="display: flex; justify-content: space-around; width: 100%;"> <span style="border: 1px solid black; padding: 2px 10px;">Help</span> <span style="border: 1px solid black; padding: 2px 10px;">Done</span> </div>																
<p>For the formula, define a variable for the travel time, and use this variable to write a rule for the distance from New York.</p>																

Figure 7a.

A hot air balloon is at an altitude of 75 feet. With time, the passengers get bored and decide to land the balloon. They descend at 6 feet per minute.

At what altitude is the balloon after 3 minutes have passed?

How high are they 7 minutes after they start to descend?

	time	altitude
Unit	minutes	feet
1.	$75 - 6 = 3$	<input type="text"/>
2.	<input type="text"/>	<input type="text"/>
Formula	<input type="text"/>	<input type="text"/>

Help Done

For the formula, define a variable for the time since they started to descend, and use  $t$  as variable to write a rule for the altitude of the balloon.

Figure 7b.

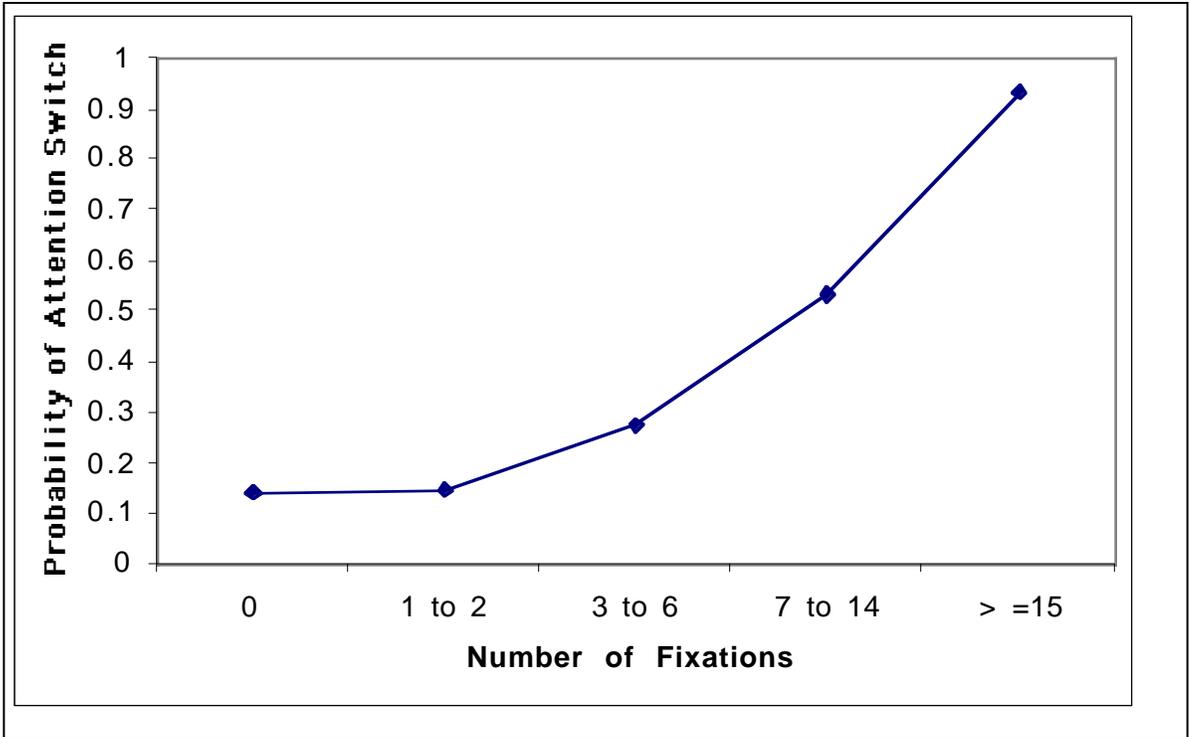


Figure 8

Figure 9a.

<p>Daily income has risen 4 dollars per year in the time since 1980. That year, the average daily income in the United States was 55 dollars.</p>	<table border="1" style="width: 100%; border-collapse: collapse; text-align: center;"> <tr> <td></td> <td style="width: 50%;">time</td> <td style="width: 50%;">income</td> </tr> <tr> <td style="text-align: right;">Unit</td> <td>years</td> <td>dollars</td> </tr> <tr> <td style="text-align: right;">Formula</td> <td><math>x</math></td> <td><math>55 + 4x</math></td> </tr> <tr> <td style="text-align: right;">1</td> <td>5</td> <td>75</td> </tr> <tr> <td style="text-align: right;">2</td> <td><input type="text"/></td> <td><input type="text"/></td> </tr> </table>		time	income	Unit	years	dollars	Formula	$x$	$55 + 4x$	1	5	75	2	<input type="text"/>	<input type="text"/>
	time	income														
Unit	years	dollars														
Formula	$x$	$55 + 4x$														
1	5	75														
2	<input type="text"/>	<input type="text"/>														
<p>Given that average, what was the daily income in 1985?</p> <p>What was the daily income in 1997?</p>																
<div style="display: flex; justify-content: space-around; margin-top: 10px;"> <span style="border: 1px solid black; padding: 2px 10px;">Help</span> <span style="border: 1px solid black; padding: 2px 10px;">Done</span> </div>																
<p>For the formula, define a variable for the time since 1980, and use this variable to write a rule for the average daily income.</p>																

Figure 9b.

The temperature drops about 2 degrees per week in Pittsburgh as time passes through the fall. The average temperature on August 24 is 83 degrees.

What is the average temperature 5 weeks after August 24?

If the temperature is 69 degrees, how many weeks have passed?

	time	temperature
Unit	weeks	degrees
Formula	$x$	$83-2x$
1	5	10*83x
2	<input type="text"/>	<input type="text"/>

Help Done

For the formula, define a variable for the time since August 24, and use this variable to write a rule for the temperature.

Figure 10a.

<p>We are currently 600 miles from Pittsburgh. Our distance from the city is increasing with time, as we drive west at an average speed of 55 miles per hour.</p>	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 10%;"></th> <th style="width: 45%; text-align: center;">time</th> <th style="width: 45%; text-align: center;">distance</th> </tr> </thead> <tbody> <tr> <td style="text-align: right;">Unit</td> <td style="text-align: center;">hours</td> <td style="text-align: center;">miles</td> </tr> <tr> <td style="text-align: right;">Formula</td> <td style="text-align: center;"><math>x</math></td> <td style="text-align: center;"><math>600+55x</math></td> </tr> <tr> <td style="text-align: right;">1</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> <tr> <td style="text-align: right;">2</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> </tbody> </table>		time	distance	Unit	hours	miles	Formula	$x$	$600+55x$	1	<input type="text"/>	<input type="text"/>	2	<input type="text"/>	<input type="text"/>
	time	distance														
Unit	hours	miles														
Formula	$x$	$600+55x$														
1	<input type="text"/>	<input type="text"/>														
2	<input type="text"/>	<input type="text"/>														
<p>You typed that in the wrong column.</p>																
<div style="display: flex; justify-content: space-around;"> <span>Help</span> <span>Done</span> </div>																
<p>For the formula, define a variable for the travel time, and use this variable to write a rule for the distance from Pittsburgh.</p>																

<p>We are currently 600 miles from Pittsburgh. Our distance from the city is increasing with time, as we drive west at an average speed of 55 miles per hour.</p>	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th></th> <th style="text-align: center;">time</th> <th style="text-align: center;">distance</th> </tr> </thead> <tbody> <tr> <td style="text-align: right;">Unit</td> <td style="text-align: center;">hours</td> <td style="text-align: center;">miles</td> </tr> <tr> <td style="text-align: right;">Formula</td> <td style="text-align: center;">x</td> <td style="text-align: center;">600+55x</td> </tr> <tr> <td style="text-align: right;">1</td> <td style="text-align: center;"><input type="text" value="20"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> <tr> <td style="text-align: right;">2</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> </tbody> </table>		time	distance	Unit	hours	miles	Formula	x	600+55x	1	<input type="text" value="20"/>	<input type="text"/>	2	<input type="text"/>	<input type="text"/>
	time	distance														
Unit	hours	miles														
Formula	x	600+55x														
1	<input type="text" value="20"/>	<input type="text"/>														
2	<input type="text"/>	<input type="text"/>														
<p>How many miles from Pittsburgh will we be in 20 hours?</p> <p>How many miles from Pittsburgh will we be in 48 hours?</p>																
<p>You typed that in the wrong column.</p>																
<div style="display: flex; justify-content: space-around;"> <span style="border: 1px solid black; padding: 2px 10px;">Help</span> <span style="border: 1px solid black; padding: 2px 10px;">Done</span> </div>																
<p>For the formula, define a variable for the travel time, and use this variable to write a rule for the distance from Pittsburgh.</p>																

Figure 10b.

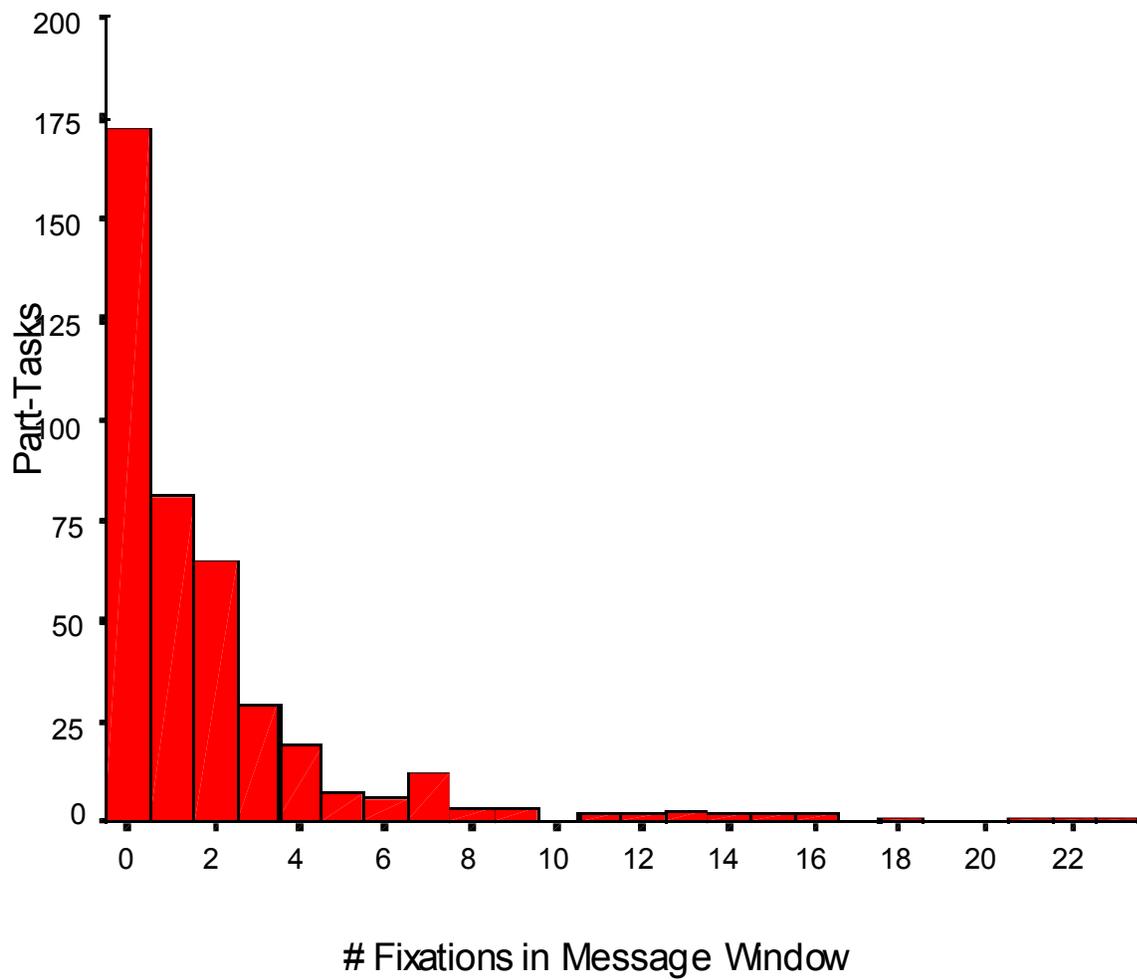


Figure 11

Figure 12.

<p>One mile equals 4 laps around the track. A jogger is concerned about the number of laps left to cover the necessary distance. He has run 12 laps so far.</p>	<table style="width: 100%; border-collapse: collapse;"> <thead> <tr> <th style="width: 10%;"></th> <th style="width: 40%; text-align: center;">distance</th> <th style="width: 50%; text-align: center;"># laps</th> </tr> </thead> <tbody> <tr> <td style="text-align: right;">Unit</td> <td style="text-align: center;">miles</td> <td style="text-align: center;">laps</td> </tr> <tr> <td></td> <td style="text-align: center;"><input type="text" value="80"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> <tr> <td style="text-align: right;">Formula</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> <tr> <td style="text-align: right;">2</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> </tbody> </table>		distance	# laps	Unit	miles	laps		<input type="text" value="80"/>	<input type="text"/>	Formula	<input type="text"/>	<input type="text"/>	2	<input type="text"/>	<input type="text"/>
	distance	# laps														
Unit	miles	laps														
	<input type="text" value="80"/>	<input type="text"/>														
Formula	<input type="text"/>	<input type="text"/>														
2	<input type="text"/>	<input type="text"/>														
<p>If he runs 20 more miles, how many total laps is that?</p>																
<p>If he runs 32 laps total, how many more miles will he have run?</p>																
<div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> <span style="border: 1px solid black; padding: 2px 10px;">Help</span> <span style="border: 1px solid black; padding: 2px 10px;">Done</span> </div> <div style="border: 1px solid black; padding: 5px;"> <p>For the formula, define a variable for the distance run, and use this variable to write a rule for the total number of laps completed.</p> </div>																



Figure 13.

<p>Daily income has risen 4 dollars per year in the time since 1980. That year, the average daily income in the United States was 55 dollars.</p>	<table style="width: 100%; border-collapse: collapse;"> <tr> <td style="width: 10%;"></td> <td style="width: 40%; text-align: center;">time</td> <td style="width: 50%; text-align: center;">income</td> </tr> <tr> <td style="text-align: right;">Unit</td> <td style="text-align: center;">years</td> <td style="text-align: center;">dollars</td> </tr> <tr> <td style="text-align: right;">Formula</td> <td style="text-align: center;">x</td> <td style="text-align: center;"><math>55+4x</math></td> </tr> <tr> <td style="text-align: right;">1</td> <td style="text-align: center;">1985</td> <td style="text-align: center;"><input type="text"/></td> </tr> <tr> <td style="text-align: right;">2</td> <td style="text-align: center;"><input type="text"/></td> <td style="text-align: center;"><input type="text"/></td> </tr> </table>		time	income	Unit	years	dollars	Formula	x	$55+4x$	1	1985	<input type="text"/>	2	<input type="text"/>	<input type="text"/>
	time	income														
Unit	years	dollars														
Formula	x	$55+4x$														
1	1985	<input type="text"/>														
2	<input type="text"/>	<input type="text"/>														
<p>Given that average, what was the daily income in 1985?</p> <p>What was the daily income in 1997?</p>																
<div style="display: flex; justify-content: space-around; margin-bottom: 10px;"> <span style="border: 1px solid black; padding: 2px 10px;">Help</span> <span style="border: 1px solid black; padding: 2px 10px;">Done</span> </div>																
<p>For the formula, define a variable for the time since 1980, and use this variable to write a rule for the average daily income.</p>																

Figure 14

<p>The oil is 30 feet deep in a storage tank on the other side of town. You measure that over time the depth is dropping at a rate of 2 feet per year.</p>	<table border="1"><thead><tr><th></th><th>time</th><th>depth</th></tr></thead><tbody><tr><td>Unit</td><td>years</td><td>feet</td></tr><tr><td>1</td><td>4</td><td></td></tr><tr><td>2</td><td><input type="text"/></td><td><input type="text"/></td></tr><tr><td>Formula</td><td><input type="text"/></td><td><input type="text"/></td></tr></tbody></table>		time	depth	Unit	years	feet	1	4		2	<input type="text"/>	<input type="text"/>	Formula	<input type="text"/>	<input type="text"/>
		time	depth													
Unit	years	feet														
1	4															
2	<input type="text"/>	<input type="text"/>														
Formula	<input type="text"/>	<input type="text"/>														
<p>How deep will the oil in the tank be in 4 years?</p> <p>How deep will the oil be in 7 years?</p>																

Help

Done

For the formula, define a variable for time, and use this variable to write a rule for the depth of the oil.

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<sup>1</sup> This research is supported by grant CDA-9720359 from the National Science Foundation and by funding from the Air Force PALACE Knight program to Kevin Gluck.

<sup>2</sup> Of course, only after repeated practice will this fact be strong enough to be successfully retrieved.

<sup>3</sup> Except for two students who have missing data. One of them has 12 part-tasks total (lost an entire day of data), and the other has 15 part-tasks (lost one problem's worth of data). In both of these cases, the data were lost because the students managed to crash the tutor mid-task, before the data were written out.