

Radical Constructivism and Cognitive Psychology

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THOSE WHO BELIEVE that education needs a foundation in the modern science of cognitive psychology sometimes feel that they are jousting with windmills. Virtually every educational movement, whatever its merits, claims to have a scientific basis. However, this is often not the case.

Unfortunately, a science of human learning has never had a large influence upon the practice of education. Until recently, such a science has not been sufficiently mature to offer much help to educational practitioners and policymakers. However, in recent decades, a body of theory and knowledge within cognitive psychology has been created that offers important opportunities for improving education. On the whole, education does not give the findings of cognitive science a large role but continues instead to struggle between two prescientific views on learning that date to philosophies of centuries past.

These two prescientific views can be characterized, at the risk of slight caricature, as follows:

The associationist philosophy holds that learning is just a matter of forming associations. Therefore, nothing problematic arises about education. All one needs to do is to teach students the associations they need to learn.

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The rationalist philosophy maintains that knowledge is to be found by looking within one's self. Therefore, nothing problematic arises about education. All one needs to do is to allow students to discover what they need to learn.

These two schools have waxed and waned in their centrality to educational practice. During periods of waxing, they acquire additional features from the current intellectual climate. The most recent waxing of the associationist school occurred during the heyday of behaviorism, under the influence of Edward Thorndike and John B. Watson, and the later influence of B. F. Skinner. At its height, this school of thought was connected to such features as programmed instruction and behavioral objectives.¹ Behaviorism has now definitely waned and has become the standard whipping boy for new reform movements in education.

One of the salient features of the behaviorist movement as a psychological theory was to reject the idea of mental structures and to assert that one could understand human thinking wholly in terms of external behavior. Behaviorism in psychology was subsequently replaced by the "cognitive revolution," which demonstrated in many ways that one could understand behavior only by postulating mental structures and processes. However, the phrase "cognitive revolution" is something of an exaggeration, for many of the paradigms and methodologies from the behaviorist era have been carried over, often much modified and augmented, to modern cognitive psychology. Thus, with major theoretical change has come cumulative progress in the science, as is typical in other sciences when they experience "revolutions."

Behaviorism, in its purest form, is applied to education by prescribing the behavior that a student should manifest and ignoring the student's thought patterns. Some educational applications in the behaviorist era maintained this theoretical purity, but often the applications were much looser and simply amounted to emphasizing immediate feedback and careful measurement of educational progress. As this approach sometimes worked well, one example of a successful behaviorist program is worth describing.

D. Porter reports on an experiment with "teaching machines" that tried faithfully to follow Skinner's principles of immediate reinforcement through knowledge of results: Teaching students to spell by means of a series of successive approximations (first the student reads the

word, then spells part of it with the other parts provided, and then spells all of it), minimizing errors, and bringing the learning situation close to the transfer situation.² In year-long training programs for the second, fourth, and sixth grades, students in the experimental groups (teaching machines) and the control groups (standard instruction) both showed about one grade-level improvement. However, students in the teaching machine groups only required one-third the class time needed by the controls to achieve this. The Skinnerian program emphasized such learning efficiency gains.

While empirical and theoretical problems with behaviorism caused it to be rejected within the science of psychology, it never was rebutted definitively as an educational program. Some projects succeeded, others failed. One approach loosely based on behaviorism was mastery-based instruction. In this approach, students were given as much time as needed to master early material before moving on to later material, to speed up learning of later material and enhance overall learning.³ Despite its generally positive empirical record, mastery-based instruction is now regarded negatively in educational circles and is not widely practiced.⁴

Behaviorist-oriented education was never a significant presence in the classrooms of America and has all but disappeared. The reasons for its demise are complex and not fully understood. The theoretical and empirical difficulties it encountered within scientific psychology and its replacement by cognitive psychology were probably only minor factors.

Recently the developments in cognitive psychology have been claimed by a new rationalist movement within education called constructivism, whose adherents overlap only slightly with the scientists who provided the experimental and theoretical content for the cognitive revolution. "Constructivism" is a vague term that covers a wide range of positions, including some that are mutually contradictory. Some versions are just attempts to bring the new theoretical insights of cognitive psychology into education.

A more extreme version, called "radical constructivism," has taken a particularly strong hold in mathematics education. (We have a particular interest in this movement because our major research concern has been learning of mathematics.) Along with the general rationalist position, it has imported pieces from two other movements that are also strongly represented in modern schools of education—situated learning

and deconstructionist critical theory. Radical constructivism emphasizes discovery learning, learning in complex situations, and learning in social contexts, while strongly distrusting systematic evaluation of educational outcomes.

Learning [in a constructivist classroom] would be viewed as an active, constructive process in which students attempt to resolve problems that arise as they participate in the mathematical practices of the classroom. Such a view emphasizes that the learning-teaching process is interactive in nature and involves the implicit and explicit negotiation of mathematical meanings. In the course of these negotiations, the teacher and students elaborate the taken-as-shared mathematical reality that constitutes the basis for their ongoing communication.⁵

This definition may be difficult to understand, so it is worth describing an instance of a successful mathematical intervention that claims a basis in radical constructivism, to match the successful intervention just cited, which claimed a basis in behaviorism. P. Cobb and his colleagues describe a second grade mathematics curriculum that embodies the principles of radical constructivism.⁶ A good example is their method for teaching second graders to count by tens. Instead of telling the students the principle directly, they assigned groups of students the task of counting objects bundled in sets of ten. Invariably, the groups discover that counting by tens is more efficient than counting by ones. Building a whole second grade curriculum around such techniques, they found their students doing as well on traditional skills as students from traditional classrooms, transferring more and expressing better attitudes about mathematics. Transfer and better attitudes are measures emphasized by radical constructivists.

It might seem a contradiction that both behaviorist and radical constructivist approaches should produce successful curricula. However, this points to the difficulty in assessing the connection between educational approaches and learning outcomes. Complex educational interventions involve change on many dimensions, making it hard to assess what features are responsible for the learning outcomes. Both interventions could have achieved their results independently of the educational philosophy ostensibly applied. More articulate theory and evaluation are needed. However, both efforts did seriously try to evaluate their interventions. Too often interventions are introduced without any real attempt at objective assessment.

Education has failed to show steady progress because it has shifted back and forth among simplistic positions such as the associationist and rationalist philosophies. Modern cognitive psychology provides a basis for genuine progress by careful scientific analysis that identifies those aspects of theoretical positions that contribute to student learning and those that do not. Radical constructivism serves as the current exemplar of simplistic extremism, and certain of its devotees exhibit an antisocial bias that, should it prevail, would destroy any hope for progress in education.

Theoretical Basis for Radical Constructivism

Frequent references are found to four sources as providing the "scientific" foundations for radical constructivism.

Modern Cognitive Psychology

Cognitive psychology is often cited as providing a basis for radical constructivism. For instance, R. Lesh and S. J. Lamson describe mathematics education largely from a radical constructivist approach:

Behavioral psychology (based on factual and procedural rules) has given way to cognitive psychology (based on models for making sense of real-life experiences), and technology-based tools have radically expanded the kinds of situations in which mathematics is useful, while simultaneously increasing the kinds of mathematics that are useful and the kinds of people who use mathematics on a daily basis. In response to these trends, professional and governmental organizations have reached an unprecedented, theoretically sound, and future-oriented new consensus about the foundations of mathematics in an age of information.⁷

This is typical of the false consensus claims that are rampant in the field. Claims are advanced in the name of modern science that have no basis in the science. In the Lesh and Lamson quote, the distinction between "factual and procedural rules" and "'models for making sense of real-life situations'" is not a distinction between behavioral psychology and cognitive psychology. They are both important theoretical components of cognitive psychology, and much current research is

concerned with which domains of thoughts are better understood in terms of "mental models" and which in terms of "mental rules."

A consensus exists within cognitive psychology that people do not record experience passively but interpret new information with the help of prior knowledge and experience. The term "constructivism" is used in this sense in psychology, and we have been appropriately referred to as constructivists (in this sense) by mathematics educators.⁸ However, denying that information is recorded passively does not imply that students must discover their knowledge by themselves, without explicit instruction, as claimed by radical constructivists. In modern cognitive theories, all acquisition of knowledge, whether by instruction or discovery, requires active interpretation by the learner. The processing of instruction can be elaborate, its extent growing with the amount of relevant knowledge the learner brings to the task.⁹

Modern cognitive psychology does not by any means enjoy agreement on all issues. Agreement has been reached on most of the basic facts of cognition and learning, while substantial controversy remains on certain matters of theoretical interpretation. Enough consensus exists today on matters of fact to support significant educational applications. To mention one in particular, the empirical evidence refutes the radical constructivists' claim that students cannot learn by direct instruction. We represent an approach to human cognition that is usually described as "information-processing psychology." Precise models of aspects of cognition provide one of its important tools, and these are often used to create computer simulations of the cognitive acts of human subjects in experiments.

Within cognitive psychology, perhaps the most controversial position with which we are associated is the "symbolic" position; that is, the claim that certain aspects of human cognition involve knowledge that is represented symbolically. In contrast, the "connectionist" position maintains that no such symbolic representations exist and that knowledge can only be described in terms of synaptic connections among neural elements.¹⁰ Controversy between the symbolic and connectionist positions has decreased since the late 1980s, and most researchers, including ourselves, have evolved (under the weight of evidence) to "hybrid" positions—recognizing that certain aspects of cognition are best understood in terms of symbolic representations and other aspects in terms of neural connections. The issue today is to decide

which aspects of cognition should be modeled in the one mode or the other, and how the symbolic and neural levels can be linked.

Some radical constructivists view themselves as opposing information-processing psychology, particularly in its symbolic form. P. Cobb, E. Yackel, and T. Wood, the most explicit on this issue, present radical constructivism as a rejection of the "representational view of mind." However, we and other cognitive psychologists, who do subscribe to a representational view, find little that is recognizable in the radical constructivists' description of that view.¹¹ Cobb, Yackel, and Wood quote R. Rorty's mischaracterization of it:

[According to the representational view of mind,] to know is to represent accurately what is outside the mind; so to understand the possibility and nature of knowledge is to understand the way in which the mind is able to construct such representations.¹²

The representational view of mind, contrary to this claim, takes into account evidence about the relation of the mind to the world and about the accuracy and completeness or incompleteness of the internal representations of the world's features. Representation, in the cognitive framework, is neither a syntactic or logical play with formal symbols lacking reference to the real world nor a literal and mechanical recording of the stimulus.¹³ In this framework, forming an internal representation of a problem situation is itself a complex psychological process.

Such radical constructivists often also limit the notion of "symbol" to verbal or logical expressions, then proceed to challenge their adequacy for modeling "nonlinear," "nonlogical," "nonverbal," or "intuitive" forms of thinking. As anyone is aware who uses a computer screen to display diagrams, pictures, or visual arrays of a great variety of kinds, no such limitations exist in the types of symbols that can be employed or in the kinds of "logical" or "nonlogical" processes that can operate on them. Considerable cognitive research recently aimed at exploring the relation of the thinking that is usually called "intuitive" with common, and well-understood, processes of recognizing familiar cues in stimuli.

A symbol (that is, a discriminable pattern) obtains its usefulness from its capacity for denoting (pointing to) objects, relations, and events in the world; but a symbolic mental representation is at best an incomplete and distorted picture of the environment, which correlates

thoughts with the information delivered by the senses, and with motor acts and their effects. Our claim is that cognitive competence (in this case, mathematical competence) depends on the availability of symbolic structures (for example, mental patterns or mental images) that are created in response to experience.

In radical constructivist writings, criticisms of the straw-man position typified by the quotation from Rorty are used to discredit the view of the mind employed in cognitive psychology. Modern cognitive theories do not assume that learning is a passive recording of experience.

Misinterpretation of the representational view leads to much confusion about the relation between external mathematical representations (for example, equations, graphs, rules, Dienes blocks, and so on) and the internal representations of these same objects. Radical constructivists, equating the representational point of view with passive recording of stimuli, without transformation, misinterpret inadequacies of the external representations as inadequacies of the notion of internal representation. For instance, if a set of rules in a textbook is incomplete, then the implication is that mental rules cannot capture the concepts. However, cognitive theories postulate (and provide evidence for) complex processes for transforming these external representations to produce internal structures that are not at all isomorphic to the stimuli. For instance, just because a diagram is two-dimensional does not mean that the mental representation of it cannot be three-dimensional.

Little in cognitive psychology supports the more extreme claims of radical constructivism. Indeed, as some radical constructivists recognize, modern cognitive psychology contradicts these claims.

Piaget

One finds frequent reference to Jean Piaget as providing a scientific basis for constructivism. Piaget has had enormous influence on the understanding of cognitive development and was one of the major figures responsible for the emergence of cognitivism from the earlier behaviorist era in psychology. While many of his specific claims have been seriously questioned, the general influence of his theoretical perspective remains. Of key importance to constructivism is Piaget's distinction between the mechanisms of assimilation and accommodation in learning and development. Assimilation incorporates experience pas-

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sively into a representation already available to the child. However, when the discrepancies between task demands and the child's cognitive structure (representation) become too great, the child must reorganize his or her thoughts. This is called accommodation (recently renamed "re-representation").

Piaget emphasized how the child internalizes knowledge by making changes in mental structure. The constructivists make frequent reference to this analysis, particularly the nonpassive accommodation process. A more careful reading of Piaget indicates that assimilation of knowledge plays a critical role in setting the stage for accommodation, that the accommodation cannot proceed without assimilation. In any event, both accommodation and assimilation are components of the representational view of mind.

Another aspect of Piaget is his stagelike characterization of cognitive development, which has led to the view that large qualitative changes occur as cognition develops. This aspect of Piaget's theory has received the least empirical support. The general view now is that cognitive changes are gradual and cumulative. The best corroborated accounts of Piagetian tasks are information-processing accounts, which identify the various components of knowledge that are being acquired.¹⁴ R. S. Siegler refers to the belief in stages as the "theory of the immaculate transition" and documents its empirical failings.¹⁵

Situated Learning and Vygotsky

One also finds frequent references to situated learning as providing a basis for radical constructivism, and one finds through situated learning references to L. S. Vygotsky. Vygotsky was a Russian psychologist in the first part of the twentieth century who emphasized the strong social character of human development. The alliance between situated learning and radical constructivism is somewhat peculiar, as situated learning emphasizes that knowledge is maintained in the external, social world; constructivism argues that knowledge resides in an individual's internal state, perhaps unknowable to anyone else. However, both schools share the general philosophical position that knowledge cannot be decomposed or decontextualized for purposes of either research or instruction.

Situated learning has become associated with the view that knowl-

edge does not transfer from the classroom to real-world situations and that instruction must take place in situations that are like the real world and often like job situations. Our critique of this aspect of situated learning led to an exchange that can be read as concluding either that situated learning is wrong in these claims or that it did not assert anything beyond the generally known fact that sometimes learning is somewhat contextualized.¹⁶ To summarize our conclusions in that discussion:

While important reforms may be needed in American education, the consensus seems to be that these reforms are not in the direction of turning the classroom into a workplace; there is merit in the powerful abstract intellectual tools that have been developed throughout human history.¹⁷

However, this exchange also converged on the conclusions that important social aspects exist to classroom learning. Beyond this, few implications can be taken from Vygotsky or situated learning for classroom practice. Little connection can be found, one way or another, between the views just discussed and the validity of group instructional methods.

John Dewey

Another figure referred to in both situated and radical constructivist writings is John Dewey. Dewey represents part of an earlier waxing of the rationalist approach to education. While Dewey started as a psychologist, he evolved into a philosopher of education, developing an educational approach loosely based on his earlier criticisms of analytic approaches in psychology before the turn of the twentieth century. Many of the ideas that are espoused in radical constructivism and situated learning are to be found in Dewey's writings, and one has an impression of modern radical constructivist educators reinventing his wheel.

Dewey started a laboratory school at the University of Chicago in 1896, and descriptions of it are enough to make a modern parent envious.¹⁸ The many progressive schools set up in the image of Dewey's Laboratory School in the first part of the twentieth century varied widely but were generally characterized by less directive instruction and more

project-oriented learning. Dewey himself was less than enthused by more radical efforts to eliminate a set curriculum from school in the name of progressive education.¹⁹

Dewey's Laboratory School is a distant memory, and many of the progressive schools based on it had almost entirely disappeared in a swirl of controversy over "life-adjustment" education.²⁰ However, each generation seems to create its own progressive schools, which emphasize many of the same features.

One of the efforts to assess the consequence of this earlier generation of progressive education for college performance found basically no differences between graduates of progressive and traditional schools in terms of their academic performance in college.²¹ A sobering notion is that radically different approaches to education resulted in few or no differences in the students who graduated.

This points to the fundamental lack of cumulative progress in education and the need to understand in more detail what is happening under banners of different educational philosophies. What aspects of the Laboratory School or other progressive schools were sound and what aspects were just fanciful anecdotes have not been determined. It was not apparent then and is not apparent now how one goes about replicating the Laboratory School in another environment. What is needed more than a philosophy of education is a science of education. Modern attempts at educational improvement point back to theorists (Piaget, Vygotsky, and Dewey) whose theories are vague by current psychological standards and lack the strong connection to empirical evidence that has become standard in the field.

Major Characteristics of the Radical Constructivist Approach

The most conspicuous characteristics of the radical constructivist approach to mathematics education are reliance on discovery learning, learning in complex "authentic" situations, learning in social contexts, and a distrust of empirical evaluations. While more than a grain of truth can be found in the suppositions motivating these approaches, they can be and sometimes are pursued to unproductive extremes.

Discovery Learning

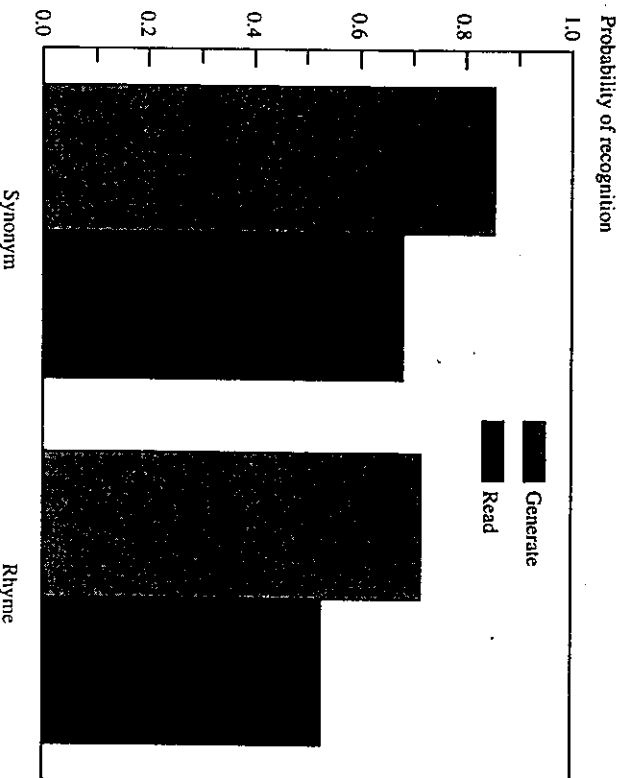
The defining feature of radical constructivism is the view that one cannot teach students but must allow them to create the knowledge that they need. This is sometimes described as a contrast between "instructivism" and "constructivism." One can readily agree that learning must be an active process, for learning requires a change in the learner, which can only be brought about by what the learner does—what he or she attends to, what activities he or she engages in. The activity of a teacher is relevant to the extent that it causes students to engage in tasks they would not otherwise undertake, including, but not limited to, acquiring knowledge provided by the teacher or by books. A teacher may also engage students in tasks, some of which may involve acquisition of skills by working examples. Other tasks include practicing skills to bring them to effective levels, interacting with fellow students, and interacting with the teacher.

The problem posed to psychology and education is to design a series of experiences for students that will enable them to learn effectively and to motivate them to engage in the corresponding activities: On both of these points, disagreement between radical constructivists and other cognitive psychologists would be hard to find. The more difficult problem is determining the desirable learning goals and the experiences that, if incorporated in the instructional design, will best enable students to achieve these goals. Arriving at good designs is not a matter for philosophical debate; it requires empirical evidence about how people, and children in particular, learn, and what they learn from different educational experiences.

A great deal of research shows that, under some circumstances, people are better at remembering information that they create for themselves than information they receive passively.²² The early study of N. J. Slamecka and P. Graf is typical of research on generative learning. They had subjects try to remember lists of words. There were four conditions:

1. Subjects generated a synonym for each word. For instance, they might be asked to generate a synonym for *ocean* that begins with *s*.
2. Subjects studied a synonym that was provided. Thus, they might study *sea* as a synonym of *ocean*.
3. Subjects generated a rhyme. For instance, they might be asked to generate a word beginning with *s* that rhymed with *tea*.

Figure 1. Probability of Recognition as a Function of Type of Elaboration, Whether Generated or Read



Source: N. J. Slamecka and P. Graf, "The Generation Effect: Delineation of a Phenomenon," *Journal of Experimental Psychology: Learning, Memory, and Cognition*, vol. 4 (1978), experiment 2.

4. Subjects studied a rhyme. For instance, they might study that a rhyme of *tea* is *sea*.

In all cases, subjects were subsequently tested for their recognition of the critical word *sea*.

Subjects learn the material more effectively in conditions where they process the meaning (synonym conditions 1 and 2) and where they generate the material (conditions 1 and 3) rather than study it passively (see figure 1). Comparing extremes, a very substantial effect emerges of 50 percent versus 85 percent recall. Cognitive psychologists debate whether this effect reflects fundamental cognitive factors or factors of motivation and selective attention.²³ However, from an educational perspective, these debates are in a certain sense irrelevant. Such effects are robust and clearly something one wants to take advantage of in instruction.

However, a number of caveats need to be made about such generation effects. First, subjects can learn in the worst conditions, and in virtually all studies, memory is not more than twice as good in generative conditions as in passive conditions. Second, experimental psychologists can only guarantee that their subjects will generate the target material by using artificial material. While the generation effect would appear to generalize to more natural material, it is difficult to guarantee that such material, because of its complexity, can be generated.²⁴ Getting students to generate much of what one wants them to learn is often difficult. In these cases, some sort of guided instruction is desirable, where one tries to induce students to generate knowledge for themselves to the greatest degree possible, but where one also provides direct instruction to maximize learning efficiency.

The argument that knowledge must be constructed is similar to the earlier arguments that discovery learning is superior to direct instruction. However, little positive evidence exists for discovery learning and it is often inferior.²⁵ Discovery learning, even when successful in enabling the acquisition of the desired construct, may require a great deal of valuable time that could have been spent practicing the construct (which is an active process, too) if it had been learned from instruction. Because most learning only takes place after the construct has been discovered, when the search is lengthy or unsuccessful, motivation commonly flags. As D. P. Ausubel wrote in 1968, summarizing the findings from the research on discovery learning:

Actual examination of the research literature allegedly supportive of learning by discovery reveals that valid evidence of this nature is virtually nonexistent. It appears that the various enthusiasts of the discovery method have been supporting each other research-wise by taking in each other's laundry, so to speak, that is, by citing each other's opinions and assertions as evidence and by generalizing wildly from equivocal and even negative findings.²⁶

Some argue that direct instruction leads to "routinization" of knowledge and drives out understanding:

The more explicit I am about the behavior I wish my students to display, the more likely it is that they will display the behavior without recourse to the understanding which the behavior is meant to indicate; that is, the more likely they will take the form for the substance.²⁷

An extension of this argument is that excessive practice will also drive out understanding. This criticism of practice (called "drill and kill," as if this pejorative slogan provided empirical evaluation) is prominent in radical constructivist writings. Nothing flies more in the face of the last twenty years of research than the assertion that practice is bad. All evidence, from the laboratory and from extensive case studies of professionals, indicates that real competence only comes with extensive practice.²⁸ By denying the critical role of practice, one is denying children the very thing they need to achieve competence. The instructional problem is not to kill motivation by demanding drill, but to find tasks that provide practice while at the same time sustaining interest.

However, experimental psychologists have shown that, under some conditions, extensive practice of material produces virtually no learning by at least some measures.²⁹ These conditions invariably exist where the experimental subjects are induced to engage in mindless recitation of the material. These results point out the grain of truth in the drill-and-kill criticisms: Students need to be engaged when they are studying.

Emphasis on Complex Learning Situations

Radical constructivists often write as if knowledge had some magical property that made it impossible to communicate and, for this reason, no simple instructional situation would suffice to convey the knowledge, whatever it might be. For example, radical constructivists recommend that children learn all or nearly all of their mathematics in the context of complex problems.³⁰ This recommendation is put forward without any evidence as to its educational effectiveness.

Two serious problems arise with this approach, given that a complex task will call upon a large number of competencies. First, a learner who is having difficulty with many of the components can easily be overwhelmed by the processing demands of the complex task. Second, to the extent that many components are well mastered, the student will waste a great deal of time repeating those mastered components to get an opportunity to practice the few components that need additional practice.

A large body of research in psychology shows that part training is often more effective when the part component is independent, or nearly

so, of the larger task.³¹ In team training, some part-task training of individuals outside the team is standard because getting the whole team together would be expensive and futile when a single member needs training on a new piece of equipment.³² In team sports, where considerable attention is given to the efficiency of training, the time available is always divided between individual skill training and team training.

There are reasons sometimes to practice skills in their complex setting. Some are motivational and some reflect the special skills that are unique to the complex situation. The student who wishes to play violin in an orchestra would have a hard time making progress if all practice were attempted in the orchestra context. However, if the student never practiced as a member of an orchestra, critical skills of coordinating with the other performers would not be acquired. The same arguments can be made in the sports context, and motivational arguments can also be made for complex practice in both contexts. A child may not see the point of isolated exercises but will when they are embedded in the real-world task. Children are motivated to practice sports skills because of the prospect of playing in full-scale games. However, they often spend much more time practicing component skills than playing games. Practicing one's skills periodically in full context is important both to motivation and to learning to practice, but not a reason to make this the principal mechanism of learning.

While motivational merit may be found to embed mathematical practice in complex situations, D. C. Geary notes that much reason exists to doubt how intrinsically motivating complex mathematics is to most students in any context. The kind of sustained practice required to develop excellence in an advanced domain is not inherently motivating to most individuals and requires substantial family and cultural support.³³ Geary argues, as have others, that this difference in cultural support accounts for the large gap in mathematics achievement between Asian and American children.³⁴

One also finds in constructivist writings an advocacy of the use of "authentic" problems.³⁵ "Authentic" is typically ill defined but with a strong emphasis on problems that students might encounter in everyday life. A focus on underlying cognitive process would suggest that this is a superficial requirement. Instead, the real goal should be to get students motivated to engage in cognitive processes that will transfer.³⁶ What cognitive processes a problem evokes is important, not what real-

world trapings it might have. Often real-world problems involve a great deal of busy work and offer little opportunity to learn the target competencies. For instance, high school mathematics classrooms where longer, more real-world-like problems were introduced to situate algebra, much student time is spent on such tasks as making tables and graphs, which rapidly become clerical in nature.³⁷ Relatively little time is spent relating algebraic expressions to the real-world situations they denote.

Reliance on Social Learning Situations

Some of the learning contexts recommended in radical constructivist writings involve tasks that can be solved by a single problem solver, but the movement more and more is to convert these to group learning situations. This undoubtedly stems in part from the influence of the situated learning movement.

The claim that instruction is only effective in a highly social environment is based on the ideas that (1) virtually all jobs are highly social in nature and (2) learning is closely associated with its context. As J. R. Anderson, H. A. Simon, and L. M. Reder have shown, the second claim is overstated.³⁸ The first claim is also probably somewhat overstated, although any analyses of job surveys that show how much social interaction, and what kind, is involved in various jobs remain unknown. Some jobs are not social in character, and this claim does not hold. Likewise, performance is highly social in other jobs. People with such jobs must learn to deal effectively with the social nature of their work.

While a person must learn to deal with the social aspects of jobs, all skills required for these jobs do not need to be trained in a social context. Consider the skills necessary to become a successful tax accountant. While the accountant must learn how to deal with clients, learning the tax code or how to use a calculator does not have to be done while interacting with a client. Training independent parts of a task separately is preferable, because fewer cognitive resources will be required for performance, thereby reserving adequate capacity for learning. Thus, learning the tax code is better without having to interact with the client simultaneously, and learning how to deal with a client is better after the tax code has been mastered.

Another facet of the claim that instruction is best in a highly social

environment comes from those claiming advantages for cooperative learning as an instructional tool.³⁹ Cooperative learning, also known as "communities of practice" and "group learning," refers to learning environments where people of equal status work together to enhance their individual acquisition of knowledge and skills. These environments are to be contrasted with tutoring (where the tutor and tutee have unequal knowledge and status) and team training (where the desired outcome is concerned with team or group performance). A review by the National Research Council (NRC) Committee on Techniques for the Enhancement of Human Performance noted that research on cooperative learning has frequently not been well controlled (for example, nonrandom assignments to treatments, uncontrolled "teacher" and treatment effects), that relatively few studies "have successfully demonstrated advantages for cooperative versus individual learning," and that a number of detrimental effects arising from cooperative learning have been identified—the "free rider," the "sucker," the "status differential," and "ganging up" effects.⁴⁰

The NRC review of cooperative learning notes a substantial number of reports of no-differences, but, unfortunately, a huge number of practitioner-oriented articles about cooperative learning gloss over difficulties with the approach and treat it as an academic panacea.⁴¹ It is applied too liberally without the requisite structuring or scripting to make it effective. Cooperative learning needs to be structured with incentives (for children at least) that motivate cooperation and a shared goal structure.⁴² The costs of this type of instruction, with uncritical application, likely will outweigh the intended benefits.

In colleges, group projects are increasingly popular among instructors, but group learning can become counterproductive. Students sometimes complain that finding meeting times for working together on assignments is difficult and that some students exploit the system by allowing other partners in the group to do all the work (and hence acquire all the knowledge and skills). A reported practice among some students is to divide the labor across classes so that one member of a group does all of the work for a project in one class, while another carries the burden for a different class. Clearly these are not the intended outcomes of cooperative learning but will occur if thoughtful implementation and scripting of the learning situation are not evident. Some of the popularity of this approach with college teachers stems from

class-size manageability: Monitoring and advising N/3 or N/4 projects is easier than *N* individual projects.

Distrust of Standard Evaluation

The denial of the possibility of objective evaluation is perhaps the most radical and far-reaching of the constructivist claims. How this principle is interpreted by all constructivists is not clear. Some radical constructivists have engaged in standard evaluations of learning interventions.⁴³ However, others are uncomfortable with the idea of evaluation, reflecting in part the influence of the philosophy of "deconstruction" on radical constructivism. D. Charney documents that empiricism has become a four-letter word in deconstructionist writings.⁴⁴ D. H. Jonassen describes the issue from the perspective of a radical constructivist:

If you believe, as radical constructivists do, that no objective reality is uniformly interpretable by all learners, then assessing the acquisition of such a reality is not possible. A less radical view suggests that learners will interpret perspectives differently, so evaluation processes should accommodate a wider variety of response options.⁴⁵

Evaluating any educational hypothesis empirically is impossible because any such test necessarily requires a commitment to some arbitrary, culturally determined, set of values. In the hands of the more moderate constructivists, the claim advocates focusing evaluation on the process of learning more than the product (what is learned), in what are considered "authentic" tasks, and involving multiple perspectives in the evaluation.

This milder perspective leads to more subjective and less precisely defined instruments of evaluation. While we share with most educators their instinctive distaste for four-alternative forced-choice questions and we agree that mathematics assessment should go beyond merely testing computational skills, we question whether the open-ended assessment being advocated as the proper alternative will lead to either more accurate or more culture-free assessment. The fundamental problem is a failure to specify precisely the competence being tested for and reliance on subjective judgment instead, with all the openings for social and intellectual bias that this reintroduces.

A number of papers addressed this issue.⁴⁶ L. B. Resnick, D. Briars, and S. Lesgold present two examples of objectively equivalent answers

(receive equal scores in their objective assessment scheme).⁴⁷ However, they are uncomfortable with this equal assessment and feel a subjective component should be added so one answer would receive a higher score because it displayed greater "communication proficiency." Although the "better" answer had neater handwriting, one might well judge it simply more long-winded than the "worse" answer. "Communication proficiency" is in the eyes of the beholder.

J. A. Dossey, in explaining the new National Assessment of Educational Progress open-ended scoring, states that a student will be given 50 percent (two points) for the right answer if the justification for the answer is "not understandable" but will be given 100 percent (four points) for the wrong answer if it "does not reflect misunderstanding of either the problem or how to implement the strategy, but rather seems to be a copying error or computational error."⁴⁸ Such subjective judgments will open the door to a great deal of cultural bias in assessment.⁴⁹ Anytime the word "seems" appears in an assessment, it should be a red flag that the assessors do not know what they are looking for. The information-processing approach would advocate specifying precisely what one is looking for in terms of a cognitive model and then testing for that.

Another sign of the radical constructivists' discomfort with evaluation manifests itself in the motto that the teacher is the novice and the student the expert.⁵⁰ The idea is that every student gathers equal value from every learning experience. The teacher's task is to come to understand and value what the student has learned. As J. Confrey writes:

Seldom are students' responses careless or capricious. We must seek out their systematic qualities which are typically grounded in the conceptions of the student. . . . [F]requently when students' responses deviate from our expectations, they possess the seeds of alternative approaches which can be compelling, historically supported and legitimate if we are willing to challenge our own assumptions.⁵¹

Or as Cobb, Wood, and Yackel write:

The approach respects that students are the best judges of what they find problematical and encourages them to construct solutions that they find acceptable given their current ways of knowing.⁵²

If the student is supposed to move, in the course of the learning experiences, from a lower to a higher level of competence, why are the

student's judgments of the acceptability of solutions considered valid? While the teacher is valued who can appreciate children's individuality, see their insights, and motivate them to do their best and to value learning, definite educational goals must be set. More generally, if the "student as judge" attitude were to dominate education, when instruction had failed and when it had succeeded, when it was moving forward and when backward, would no longer be clear.

Understanding why the student, at a particular stage, is doing what he or she is doing is one thing. Helping the student understand how to move from processes that are "satisfactory" in a limited range of tasks to processes that are more effective over a wider range is another matter. As L. B. Resnick argues, many concepts that children naturally come to (for example, that motion implies force) are not what the culture expects of education and in these cases "education must follow a different path: still constructivist in the sense that simple telling will not work, but much less dependent on untutored discovery and exploration."⁵³

A Cognitive Psychology Alternative

While we cannot claim to possess a philosophy that specifies all the answers to how education should proceed, modern cognitive psychology does contain some pointers about how one should progress in mathematics education.

Cognitive Task Analysis

If a single central theme can be pointed to in research in cognitive psychology, then it is that conceptual power derives from taking a complex cognitive phenomenon and analyzing it into its underlying components; that is, understanding the behavior of the whole from an understanding of the components and their interactions. Debate continues within the field (and research to settle such debate) as to what the components are, but general agreement has been reached that more understanding results from such a task analysis. Often different theoretical proposals for the components turn out to have similar consequences, because these proposals are still analyzing the same task, and

the structure of that task is crucial.⁵⁴ What is important is to analyze what the task structure means for the mind of the person performing it.

In the context of education, real value is found in identifying the components that a student needs to learn and targeting their instruction. In many cases, this knowledge is not apparent in the surface structure of a problem, and students have difficulty learning as a consequence. A major problem, for example, in the acquisition of geometry proof skills was that students had difficulty identifying the component skills.⁵⁵ Typically in geometry, students are shown complete proofs and are left to figure out what problem-solving steps underlie finding these proofs. A similar problem seems to haunt students' attempts to master algebra word problems.⁵⁶ Task analysis has played an important role in efforts to teach mathematics in American schools and in Chinese schools.⁵⁷

Task analysis will often reveal prerequisite knowledge required for students to learn a new competence. Often this prerequisite knowledge has not been mastered by significant subsets of the student population. An example is knowledge of the number line and basic operations on it. R. Case and S. Griffin found that many at-risk students lacked this knowledge, which is a prerequisite to mastering early school mathematics.⁵⁸ Explicitly teaching this knowledge to the students dramatically increased their success at first grade mathematics.

As knowledge domains become more advanced, their underlying cognitive structure tends to become more obscure. Thus, while providing feedback on the final answer remains easy, providing feedback on the individual mental steps that lead to the answer becomes difficult. Teachers often are unaware, at an explicit level, of what this knowledge is and do not know how to teach it. Attempts to convey some relatively basic skills reveal this problem. A good case in point is reading when one goes beyond the basic word identification skills. A. S. Palincsar and A. L. Brown were able to produce dramatic improvements in students' comprehension skills by introducing to the students and then having them practice the skills of summarizing, clarifying difficulties, asking questions, and so on.⁵⁹ These are valuable activities to engage in while reading, but apparently these children had not been taught them, and they were unable to acquire them independently.

Analysis of incorrect performance by students is also valuable for revealing systematic errors or bugs in their thinking. In many cases, not much feedback occurs during the typical conditions of practice, and the

student might wind up entrenching the wrong knowledge structures. A well-studied case of this is subtraction, where students can acquire the wrong rules and practice them to a state of perfection.⁶⁰ A comparable situation involves naive physics; students may have spent a lifetime of practicing the wrong physics, which is hard to discard when they come into the classroom.⁶¹ Making teachers aware of the systematic confusions that students suffer produces improved educational outcomes.⁶²

Role of Deliberate Practice

One unfortunate consequence of the popular slogan "drill and kill" is that it leaves the impression that practice is bad. On the contrary, studies of expertise have thoroughly established that expert skills take a long time to acquire and a great deal of practice. Cognitive task analysis shows why. Underlying any complex competence are a large number of knowledge components, each requiring substantial practice to be mastered. J. R. Anderson has shown that learning a complex competence requires learning its many components.⁶³

A conscious effort almost seems at work to discredit the importance of time spent learning. C. C. McKnight and others try to debunk this factor.⁶⁴ They report statistics for seventh grade Japanese and eighth grade American students receiving 101 and 144 hours of instruction, respectively. However, this comparison is misleading. The seventh grade is the only year in twelve years of schooling that Japanese students spend so little time in mathematics. They spend 175 hours in most of elementary school (where the class time devoted to mathematics is twice that of American students), 140 hours in the rest of junior high, and more hours again in senior high.⁶⁵ In addition to, but not included in, these tallies are the numerous hours many Japanese students spend in after-school classes called *juku* where they get further intensive tutoring. Moreover, time is spent much more efficiently in the Japanese classroom. Students receive instruction 90 percent of the time in a Japanese classroom but only 46 percent of the time in an American classroom.⁶⁶ In the Pittsburgh public schools, despite an official figure of 140 hours, ninth grade students average only about sixty-five hours a year learning mathematics in the classroom.

Psychological studies of learning show repeatedly that the first variable of human learning is time on task. Granted, other factors, such as

how one spends that time, matter. Also, one could spend time learning useless things. But amount learned is roughly proportional to amount of time spent learning.⁶⁷ The second variable of human learning (that is, forgetting) is time away from task. Thus, the long summer vacation in America becomes suspect, and dropping some important topics such as algebra for a year (typically geometry intervenes between algebra I and algebra II) creates ideal opportunities for forgetting to do its work. Teachers frequently complain about all the reteaching they have to do after summer vacation.

While time on task is critical, rote drill is not advocated. How time is spent is critically important. As constructivists (and others) emphasize, one wants students to be actively involved in the learning process. K. A. Ericsson, R. T. Krampe, and C. Tesch-Römer, in their study of the development of expertise, emphasized "deliberate practice."⁶⁸ Deliberate practice is defined as involving motivated subjects receiving informative feedback with careful and continuous coaching and monitoring. Unfortunately, these conditions are often not met in American classrooms. H. W. Stevenson and J. W. Stigler have emphasized the involved and focused explanations and discussions that are part of Asian mathematics classrooms.⁶⁹

As important as practice is for the student, it is important for the teacher to practice, too. Asian teachers spend more time than American teachers preparing and practicing their instruction. Stigler and Stevenson state that Japanese teachers give what amounts to a performance in a mathematics classroom.⁷⁰ American teachers teach smaller classes but have fewer preparation periods, while Asian teachers teach larger classes but also receive more preparation periods. One thing that facilitates the development of teaching expertise in Asian schools is that the curriculum is relatively constant and does not change much from year to year. In contrast, in the reform-minded and faddish world of American education, the curriculum never stays constant long enough for teachers to reach a level of mastery.

Transfer, Insight, and Understanding

Radical constructivist writings often recommend that students learn "with understanding" in contrast with the old "behaviorist" approaches of rote learning. Determining what "understanding" might

mean in learning and its application to real tasks is vital. Understanding a concept means nothing more nor less than having a rich network of knowledge structures that can be used to solve problems that involve the concept flexibly in many contexts. Each one of the knowledge structures has to be learned separately. Thus, understanding of a domain does not come in one fell swoop of insight but is built up bit by bit over time.

For example, to say that a student has understood a concept such as fractions means that the student can use that knowledge flexibly in many situations. Thus, the student can figure out how much pizza each of three children will get if they have to share half a pizza; the student will recognize that, when thirty-five people must be transported by buses that each hold twenty people, two buses are required, not one-and-three-quarters; the student can explain why one inverts a fraction to divide by it; and so on. A child does not suddenly acquire the ability to do all of this.

The belief in moments of transformation in education is undoubtedly linked to the old belief in developmental psychology that children transit abruptly between stages, which in turn is often linked to Piaget. Instead, as R. S. Siegler documents with great care, development is always gradual and continuous.⁷¹ The same is true of education.

One classic contrast between learning with understanding and rote learning is M. Wertheimer's comparison of students taught to solve problems by rote or "insight."⁷² Students given insight into the formula for the area of a parallelogram (by observing a construction) were able to transfer it to other figures for which the formula (base times height) is correct. Children just taught the formula were not able to transfer this knowledge. Children in both conditions learned a set of facts and procedures. However, in the insight condition, they were taught a different and richer set of facts that enabled the transfer. The insight instruction took longer, reflecting the richer knowledge that was learned.

M. K. Singley and J. R. Anderson studied extensively the conditions under which knowledge learned in solving one kind of problem would transfer to solving another kind of problem.⁷³ They showed that transfer between domains was typically not all-or-none but varied with the amount of knowledge the two domains shared. Understanding how knowledge will transfer between domains depends critically on task analyses that examine the knowledge structures that the learner has

acquired in one domain and assess their applicability to another domain. Transfer will occur to the extent of shared cognitive elements.

Writers on education who argue for magical moments of understanding in which knowledge becomes transformed point to the phenomenon of insight. Some experimental research has been done on insight problems.⁷⁴ One striking feature of such problems is that subjects do not know that they are close to producing a solution much before they arrive at it. Because insight problems are defined as problems that require a single key insight for their solution, not surprisingly it only takes a little time to encode that one bit of knowledge once it is recognized. In contrast, noninsight problems (such as doing a proof in geometry) require developing multiple pieces of knowledge. In such cases, students can judge when they have solved some, but not all, of a problem.

In a careful analysis of the mutilated checkerboard problem, a famous insight problem, C. A. Kaplan and H. A. Simon studied the relation between the critical insight and the rest of the problem solution.⁷⁵ The mutilated checkerboard problem requires deciding whether it is possible to cover a checkerboard with dominos that each cover exactly two squares of the board. When two squares are cut out from opposite corners of the checkerboard, covering the whole board becomes impossible, because each domino covers a black and a white square and two opposite-corner squares of the same color have been removed. This is called the parity insight; and subjects typically spend several hours trying unproductive paths before considering it, but then solve the problem relatively rapidly. However, despite the apparent sudden nature of this solution, steps lay the foundation for the insight, such as choosing to consider invariances in the problem. Moreover, when subjects did think of the insight, they still had to go through the process of working out its implications. Complete proofs of impossibility did not occur instantaneously but had to be developed given the decision to regard parity. So, even in insight problems, bundles of knowledge do not come magically in one fell swoop but have to be worked out piece by piece.

Empirical Assessment

More than anything else, the development of educational methods needs constant assessment. Research programs should not be foisted on

the public as educational programs until a careful analysis has been conducted of their learning consequence. A need exists for something analogous to the Food and Drug Administration, which would assess the consequences of educational programs just as the consequences of drugs are assessed before releasing them. Most fad educational movements would not survive such empirical evaluations. Would resistance emerge to empirical validation within the radical constructivist movement if no fear arose of what the evaluations would show? Commitment to a philosophy of education encourages unwillingness to have the philosophy disproved.

Part of the difficulty that radical constructivists, and other educators as well, have with assessment is that no evaluation instrument is perfect. To focus on the deficiencies is to ignore the information that an assessment provides. A classic example was the assessment of Project Follow Through, which found that direct instructional methods were more effective in the early grades with at-risk children of low socioeconomic status than open classroom methods.⁷⁶ This report was immediately drowned in criticisms of all of its shortcomings, completely distracting attention from the important information that was contained in the evaluation.⁷⁷ Scientists know that they must be sensitive to the limitations of their instruments but that they must not ignore what their instruments are saying.

While knowing which educational perspectives produce positive results is important, also of significance is understanding what aspects of the intervention are producing the various parts of the learning outcome. Within research on educational evaluation, a distinction is made between "summative" evaluations and "formative" evaluations. Summative evaluations try to identify the learning consequences of some fixed educational treatment, while formative evaluations try to identify how a treatment can be improved. More research is needed in cognitive psychology that would perform a formative role and help to determine what works and what does not work in educational applications. Regrettably, much research in cognitive psychology is too abstracted from the real world to be of use to the educator. More laboratory research needs clear connections to educational problems so that it could better inform educational interventions.

Important issues of empirical assessment do exist. A serious one is to define goals. No instructional intervention will optimize everyone's

goals. Furthermore, different assessments may reach different conclusions about what a student has learned. Therefore, the goals of education must be discussed and the consequences of different methods of assessment must be considered. However, these legitimate concerns should not cause educators and scholars to ignore gathering those data that can inform their thinking.

Conclusion

The time has come to abandon philosophies of education and turn to a science of education. Consider the analogy of medicine. For thousands of years, before any real knowledge of human physiology existed, remedies for some pathological conditions were known and used, sometimes effectively, by both doctors and others. J. Gleick has provided a vivid description of prescientific medicine:

Its practitioners wielded the authority granted to healers throughout human history; they spoke a specialized language and wore the mantle of professional schools and societies; but their knowledge was a pastiche of folk wisdom and quasi-scientific fads. Few medical researchers understood the rudiments of controlled statistical experimentation. Authorities argued for or against particular therapies roughly the way theologians argued for or against their theories, by employing a combination of personal experience, abstract reason, and aesthetic judgment.²⁸

When medicine began to adopt the methods of science, far more powerful treatments were developed concurrently with the development of modern physiology and biochemistry; treatments are now based squarely on these sciences. To acquire powerful interventions in disease, understanding of the mechanisms of disease—of what was going on in the diseased body—had to be deepened. This is the revolution of twentieth century medicine, and its results speak for themselves.

In the same way, human beings have been learning, and have been teaching their offspring, since the dawn of the species. A reasonably powerful "folk medicine" has evolved, based on lecturing and reading and apprenticeship and tutoring, aided by such technology as paper and the blackboard—a folk medicine that does not demand much knowledge about what goes on in the human head during learning and that has not changed radically since schools first emerged.

To go beyond these traditional techniques, the example of medicine must be followed and (as cognitive psychology has been doing for the past thirty or forty years) a theory of the information processes that underlie skilled performance and skill acquisition must be built. A theory is needed of the ways in which knowledge is represented internally, and the ways in which such internal representations are acquired. Cognitive psychology has now progressed a long way toward such a theory, and much is already known that can be applied, and is beginning to be applied, to improve learning processes.

If progress is made to a more scientific approach, traditional educational philosophies will be found to be like the doctrines of folk medicine: They contain some elements of truth and some elements of misinformation. This is true of the radical constructivist approach. Only when a science of education develops that sorts truth from fancy—as it is beginning to develop now—will dramatic improvements in educational practice be seen.

Comment by K. Anders Ericsson

Important implications of the training of expert performance exist for general education in the schools and for particular educational methods advocated by proponents of radical constructivism.

The greatest scientific advantage of education of expert performers compared with general education is that the final goal for training experts is specified from the start and agreement has been reached on how to assess the attained level of performance for individual subjects and thus to evaluate the outcomes of training. In contrast, the goal of general education in public schools is much broader and needs to include the successful preparation for many different occupations and obligations of citizens. Consequently, beyond the common goals of general education, such as the fostering of independent, creative, and productive members of society, identifying specific educational goals that are uniformly valued and can be objectively assessed has been difficult. A further challenge is that society is going through dramatic structural changes, so determining which specific skills and knowledge that will be essential ten to twenty years from now is hard. Hence, modern

educators have trained many generalizable abilities such as creativity, general problem-solving methods, and critical thinking. However, decades of laboratory studies and theoretical analyses of the structure of human cognition have raised doubts about the possibility of training general skills and processes directly, independent of specific knowledge and tasks. For example, research on thinking and problem solving show that successful performance depends on special knowledge and acquired skills, and studies of learning and skill acquisition show that improvements in performance are primarily limited to activities in the specific domain.⁷⁹ Some recent theoretical approaches to education can be viewed as direct reactions to the lack of generalizability of traditional education in, for example, mathematics and science.

Proponents of radical constructivism argue that an important reason for the failure of contemporary instruction lies in its implicit encouragement of memorization and mindless drill.⁸⁰ Students find memorizing information by rote easier than understanding the studied information and relating it to relevant experiences in their lives. Today's teachers are said to have structured the learning activities so much that students are more successful by guessing what the teachers want to hear than by trying to generate the correct answers by careful thought. The proposals by radical constructivists to remedy this educational failure rely on a return to the learning processes underlying natural cognitive development. Greatly influenced by Jean Piaget's theories of assimilation and accommodation, radical constructivists argue that learning evolves from the necessary adaptations required by successful engagement in activities in the task domain.⁸¹ Genuine learning is self-directed, and many radical constructivists agree with Piaget that "each time one prematurely teaches a child something he could have discovered for himself the child is kept from inventing it and consequently from understanding it completely."⁸² Finally, the inherent enjoyment of engaging actively in reasoning and problem solving can be fostered only if the students generate or chose the problem as their own.⁸³ Based on these considerations, radical constructivists recommend educational settings where students are forced to take the initiative and guide their own learning. Many radical constructivists even discourage the teacher from correcting students when their reasoning and ideas are invalid because such criticism may jeopardize their self-confidence in their independent reasoning and challenge their self-respect.⁸⁴ In sum, radi-

cal constructivists believe that self-guided learning will lead to genuine understanding and to skills for independent thinking and reasoning.

In their criticism of the educational method of self-guided learning, John R. Anderson, Lynne M. Reder, and Herbert A. Simon argued convincingly that recent advances in cognitive psychology and cognitive science provide educational methods that are superior to self-guided learning.

Experience and Expert Performance

Recent reviews show that extended engagement in activities of a chosen domain is necessary to attain expert performance.⁸⁵ First, when performance is assessed with representative measurement criteria in longitudinal studies, no evidence is available for sudden increases in performance from one time to the next. Second, expert performers continue to improve their performance beyond the age of physical maturation (the late teens in industrialized countries) for many years and even decades. The age at which performers typically reach their highest level of performance of their career is in their mid-to-late twenties for many vigorous sports and for arts and science a decade later or in their thirties and forties. The continued, often extended, development past physical maturity shows that experience is an essential factor mediating improved performance. The most compelling evidence for the necessity of vast experience before attaining high levels of performance is that even the most talented need around ten years of intense involvement before they reach an international level.⁸⁶ The necessity of active engagement to improve performance in a domain of expertise is well established and consistent with claims by both radical constructivism and the information-processing theories of learning and skill acquisition.

When individuals initiate regular engagement in a domain, whether a type of leisure or work, they go through a limited period of relatively rapid improvements when salient errors are corrected until they reach an acceptable level of performance. However, after that initial improvement, further increases are typically small. More generally, the length of experience in the domain has been found to be at best a weak predictor of current level of performance in a wide range of domains.⁸⁷ Hence, mere engagement in activities (experience) does not lead to improvement in performance.

The lack of benefit of additional experience on improved accuracy of performance is consistent with Piagetian notions of accommodation endorsed by radical constructivists and notions of impasse as a condition for changes to the mediating cognitive mechanisms.⁸⁸ When individuals' engagement with their environments runs smoothly, no change in the structure of performance would be expected. Even if the activity fails, no opportunities typically arise for corrections. Under these types of conditions, observable improvements of performance would hardly be expected. However, the same performance could easily be improved by special training activities designed to improve performance. The training involves the design and presentation of situations that challenge the trainees sufficiently but that they can master with full concentration and repetitions. The term "deliberate practice" has been used to refer to training activities that were designed by a teacher solely for the purpose of improving an individual's performance.⁸⁹ To engage in these training activities is judged to be effortful and less enjoyable than regular recreation, and thus active participants in domains rarely engage in deliberate practice even though they recognize that engaging in it would improve their performance. In sum, active engagement in a domain does not invariably, nor even typically, lead to improvement of performance, once some initial acceptable adaptation has been attained.

In almost every domain, promising individuals are supervised by a teacher who instructs them and designs their practice from a very young age. B. S. Bloom found that many international-level performers had relocated to be close to a desired teacher or an excellent training environment and virtually all of them had sought out a teacher, who either had reached the international level or had prior students who had reached that level.⁹⁰

Several factors make it nearly impossible for individuals to guide themselves to expert levels of performance without the help of excellent teachers. Whereas cognitive development in children is surprisingly invariant across two different environments and cultures—that is, African rural and American suburban communities—large differences can be observed in domains of expertise, such as music, sports, and science, that depend on the historical time and the specific culture.⁹¹ One of the primary reasons is that domains of expertise have over time developed methods for accumulating discovered knowledge, skills, and produced artifacts and hence extracted an externalized body of organized expe-

rience that can be transferred from the current to the next generation.⁹² Individuals no longer must discover knowledge and methods from scratch, and individuals are not only able to match but also to surpass the level attained by pioneering predecessors. The necessary role of teachers for mastering any of the arts and sciences becomes apparent when one considers that the accumulation of knowledge and achievements is based on specific shared concepts, notational systems, and instruments and that the innovation of new concepts and laws was a prerequisite for the emergence of modern theories and their highly efficiently organized knowledge. The increases in level of expert performance over time are taken for granted in science and sports, but the improvements in instruments and equipment makes inferences about large changes in skill level difficult. However, in domains with fewer changes in instruments, such as performance with the piano and violin, today's performers readily master music that was considered unplayable by the best musicians in the nineteenth century and can match or often surpass the technical virtuosity of legendary musicians of the past.⁹³ Similarly, in many sports with minimal equipment, such as running, diving, and swimming, the highest level of performance attained early in the twentieth century is now commonplace and matched by a vast number of serious amateurs.⁹⁴

In all major domains, an accumulation of effective methods has occurred for teaching the accumulated knowledge and skills. Over the last couple of centuries, teachers and coaches have gained insights into how sequences of easy training tasks can allow students to eventually master more complex tasks, which may often at first sight appear unattainable to the student. Furthermore, teachers know how and to what degree of mastery the simpler tasks have to be acquired to serve as building blocks of more complex skills. Unlike the beginners themselves, teachers can foresee the future demands and avoid the need for complete relearning of previously attained skills. The core assumption of deliberate practice is that expert performance is acquired gradually and that effective improvement of students' performance depends on the teachers' ability to isolate sequences of simple training tasks that the student can sequentially master by repetition with feedback and instruction.⁹⁵ Deliberate practice requires training tasks with a difficulty level such that they lie outside the students' current repertoire, and their mastery requires that the students concentrate on critical aspects and

gradually refine their performance through repetition in response to feedback. Hence, the requirement for concentration sets deliberate practice apart from both mindless drill and playful engagement as the latter two types of activities would, if anything, merely strengthen the current structure of the performance rather than change it.

In many domains, promising children start training with teachers at very young ages. Because of the requirement of sustained concentration, the duration of training is initially short—typically no more than fifteen to twenty minutes per day, which leaves room for other activities. Many parents help their children to concentrate during practice, to establish regular practice patterns, and to encourage them by pointing out practice-related improvements in their performance.⁹⁶ With increasing age, the involvement of future expert performers increases, and toward the end of adolescence the commitment to the domain-related activities is essentially full time. Furthermore, recent reviews have assembled a broad range of evidence supporting the claim that individual differences in giftedness and talent, especially among children, can be attributed to differences in practice history rather than any innate differences in talent.⁹⁷ More talented children improve faster in large part because they spend more time in practice each week.⁹⁸

Cognitive Mediation and Internalization

When individuals try to master an everyday activity, such as driving a car or typing, the goal is typically to achieve effortless performance as rapidly as possible. After some limited period of training and experience, the appropriate responses are elicited by the individuals without the need for effortful attention, and the skill has been automatized.⁹⁹ In contrast, the key challenge for aspiring expert performers is to avoid the arrested development associated with automaticity and to acquire skills to support continued learning and improvement.

The superior performance of experts can be reliably reproduced in laboratories by giving them representative tasks that capture the essence of their expertise.¹⁰⁰ This general approach was originally proposed by A. de Groot, who instructed good and world-class chessplayers to think aloud while selecting the best move to the same set of unfamiliar chess positions.¹⁰¹ He found that the quality of the selected moves was closely

associated with chess skill. In a recent review, K. A. Ericsson and A. C. Lehmann found that in a wide range of domains experts' think-aloud protocols revealed how their superior performance is mediated by deliberate preparation, planning, reasoning, and evaluation.¹⁰² At increased levels of performance, individuals have acquired improved mental representations to maintain accessibility to relevant information and to support more extensive and flexible reasoning about an encountered task or situation.¹⁰³ In most domains, better performers are able to rapidly encode and store relevant information for representative tasks.¹⁰⁴

The training of future expert performers is not only a matter of shaping and increasing their performance but also involves the acquisition and refinement of mental representations that allow the student to image desired performance and to monitor concurrent performance. When beginners are initially introduced to practice in a domain, the teacher presents them with simple objectives and tasks and will often explicitly instruct the beginners to pay attention to specific aspects. The assigned goal for the training activity provides the beginners with feedback, which is supplemented by the teachers' instruction to the subjects to make specific changes and corrections. As the complexity of the acquired level of performance increases, so does the complexity of the practice tasks and goals. The teacher will primarily provide higher level instruction that requires that students are able to monitor their performance and are able to engage actively in problem solving to correct errors and improve performance. Hence, in parallel to improvement of performance, students acquire improved representations to image the desired performance, to monitor their performance, and to identify methods to reduce discrepancies.¹⁰⁵

In many instances, the relevant information that is extracted and encoded changes as a function of attained level of performance. For example, the primary reason that expert racquet players can react rapidly to return a fast serve is not the result of an innate speed advantage but the acquisition of improved perceptual skills to anticipate the path of the ball.¹⁰⁶

The highly developed representations allow skilled performers to improve by training organized by themselves. One of several general methods consists of the study and analysis of the performance and achievements of masters in the field.¹⁰⁷ Self-study allows for gradual

refinement of representations and associated knowledge through attempts to reproduce the achievements of masters in the domain.

Creativity

The popular view of creativity still holds that the creative process is spontaneous and reflects a highly personal contribution from mysterious sources. Within this framework, creativity is believed to be higher during childhood and increasingly stifled by education and the demands for deliberate practice. In contrast, the expert-performance view focuses on providing the students with all the tools that give them the necessary control to image and create their products and achievements. The highest level of achievement in a domain, according to this view, involves the making of a major creative contribution that changes or redefines the boundaries and definition of that domain of expertise. To have a chance to make a major innovation, performers must have assimilated the prior relevant knowledge and be familiar with earlier related achievements. Extended education is, thus, necessary even to recognize a major innovation as such if it were to be generated or encountered accidentally.

The empirical evidence on creative achievement shows that individuals have not, as a rule, been able to make creative contributions to the domain until experiencing a long preparatory period during which they mastered the relevant aspects of that domain. Even in the cases of revolutionary innovation where the creative ideas redefine the domains, the creative individuals have a long history of education during which they studied and mastered the existing techniques, such as Picasso.¹⁰⁸

In sum, the training of expert performers should not stifle creativity but instead provide the tools and knowledge to empower the experts to be more successful and effective in their daily work and their search for innovative ideas, especially those rare ones that go beyond the current knowledge and practices. Given the unpredictable nature of innovations, their generation must reflect some type of playful exploration of possibilities.¹⁰⁹ Hence, the extended education of expert performers primarily elevates the level of play by providing the appropriate tools and the rich knowledge of other experts' previous achievements.

Conclusion

In many different domains dedicated to optimal development of expert performance, a fairly similar pattern of training has evolved over decades or centuries of experimentation. Consistent with the proposal of radical constructivism, the focus is on keeping the student actively engaged in activities. However, rather than encouraging the students to select their activities by themselves, teachers guide the individual development of students and design their training and monitor progress. The designed training activities help the student focus on selected goals and provide feedback and opportunities for refinement by repetition. Throughout the extended training, the future experts acquire improved representations that allow them to guide and monitor their own learning to prepare for independence as performers and as guides for their future development. When experts have assimilated the accumulated relevant knowledge in their domain and have all the necessary tools, the education is completed and the expert goes on independently to assimilate new knowledge and make individual creative contributions. Hence, the controversy over how much and whether teachers should guide the development of mastery of domains of knowledge is one of determining at which point students have acquired the necessary mental tools to design and monitor their own learning effectively and to reach specific standards of mastery. These assessments must be made on a case-by-case basis, but the general implication from studies of expert performance is that the refinement of representations that support reasoning and understanding continue for many years or even decades of adulthood.

Some more general implications can be drawn for educational practices from studies of expert performance and deliberate practice. They show how mastery is learned and that everyone improves his performance by focused training. The more that is understood of the cognitive representations and complex skills that mediate expert performance and continued improvement of performance, the more that is understood of the educational challenges of facilitating their acquisition. With new methods of assessment, educators should be able to monitor the acquisition of the representations, give feedback of the progress, and propose remedial training activities, if necessary. Most important, the complexity of the mechanisms mediating expert performance shows that these

mechanisms would not result from mindless drill with any training tasks. Master teachers and expert performers unanimously claim that during deliberate practice individuals have to maintain full concentration and be actively engaged to modify their performance.¹¹⁰ Master teachers recommend that the students learn to monitor their level of concentration and, when it cannot be sustained fully, that they stop their practice to recuperate.

The real challenge for educators is that effective improvement of performance requires active engagement and concentration by students. The challenge is similar in domains of expertise where individuals are drawn to the inherent enjoyment of playful social interactions. In contrast, parents and teachers almost invariably have to actively support the engagement in deliberate practice and show its instrumentality in attaining the desired higher level of performance. One possibility would be to try to convert practice into play. However, when educators propose to remove guidance and feedback from learning activities, one might worry that these more playful activities may be more enjoyable but at the direct expense of their effectiveness in improving performance.

The option that emerged through the long history of training of expert performance involves the increase of learning effectiveness with designed training activities that require concentration. With extended experience, knowledge of how to best schedule and motivate practice has been accumulated, but the most important insight concerns the limits on daily deliberate practice and the need for relaxation and recuperation. When individuals start practice, the daily duration of deliberate practice is short (fraction of an hour). Although it increases with the number of years of training, not even full-time professionals appear to be able to sustain more than four to five hours of deliberate practice every day without risking exhaustion and eventual burnout. Designed training activities and deliberate practice provide powerful tools for efficient learning in the schools. However, teachers must help their students to learn to monitor their level of concentration and also arrange a flexible curriculum that mixes deliberate practice with alternative activities that require less concentration and opportunities for relaxation. More generally, these and other insights from the training of experts will contribute information about the constraints and potential for effective learning in the public schools.

Comment by Robert Glaser

Distinguished cognitive psychologists, in an important step for science and society, are now engaged in the application of principles of human cognitive performance and learning to improving education. Not to be neglected is the integration of knowledge of human mentality with professional knowledge and educational practice. The question is: How can what is known about cognition, about the environments in which cognitive processes are nurtured, and about the details of high levels of competence be used to maximize the abilities of U.S. students? Toward this end, appropriate tactics must be learned from the ventures of other sciences in practical developments where the interaction between science and practice has been enormously beneficial for both. For example, the press for the development of transistors, space flight, and health and physical well-being have opened up new fields of science and service. So, too, can educational design reap great benefits from the current engagement of top-notch scientists interested in cognition, culture, and human development.

At the beginning of my interests in this work, two major events stimulated interaction between theory and educational practice. One, mentioned by John R. Anderson, Lynne M. Reder, and Herbert A. Simon, was the movement of B. F. Skinner's behaviorist psychology into the educational scene. Within a decade, hundreds of instructional programs were published, different kinds of teaching machines were for sale, and societies were founded in a dozen countries. The reasons for the demise of this movement are left as a puzzle by Anderson, Reder, and Simon. To my mind, there were two: individualization of the rate of instruction and mastery criteria made the conventional school structure unmanageable, and behavioristic theory did not have the principles for a cognitive analysis of performance and could not go the distance in attending to learning through understanding and reasoning. From the point of view of the interaction between science and practice, in programmed instruction's rush to be put to use, applications were quickly separated from any test of the theory underlying them. A mutually correcting system in which failures and limitations in both practice and theory could be confronted never developed.

The second more powerful event in the interaction of application and

theory was that World War II aroused interest in research on complex human performance, much of which focused on the performances involved when individuals controlled complex systems of people and machines, generally concerned with the detection and transfer of information required for decisionmaking. A link was forged between human cognitive capacities and models of performance of these capabilities in terms of information processing systems. Here, practical necessity contributed significantly to modern cognitive theories of human performance.¹¹¹

A modern science of cognition developed that influenced and was influenced by the study of cognitive development in children. L. B. Resnick and I noted in the 1972 *Annual Review of Psychology* that "in increasing numbers, experimental psychologists are turning their enterprise to analyses and investigations of the instructional process."¹¹² Over the past twenty-five years, renewed study of learning and the interaction of theory and practice has taken place. The opportunity was offered to interpret advances in cognitive psychology in either constructive or unconstrained ways.

Advances in Cognition and New Conceptions of Learning

Some brief examples of areas of study that consider the interaction of learning theory and educational design are: (1) the analyses of functional, proceduralized knowledge, (2) metacognitive self-regulatory abilities, and (3) the access to knowledge afforded by cultural experiences and community practices.

The first area is functional knowledge and skill. The study of memory has moved beyond the behavioristic theories of simple associations to the descriptions of coherent structures that represent knowledge and meaning. The integration of knowledge behind human performance is now represented by larger, more organized constructs that explain the power and speed of mental activities. And modern learning theory is faced with the challenge of delineating the conditions that assist in establishing structure and coherence in acquired knowledge.

Toward this objective, studies of learning and studies of the differences between beginners and competent individuals indicate that a course of knowledge acquisition proceeds from a declarative or prop-

ositional form to a compiled, effectively used form.¹¹³ Novices can know a principle, a rule, or a specialized vocabulary without knowing the conditions of effective application. In contrast, when more expert learners access knowledge, it is functional in the sense of being bound to conditions of use. Experts and novices may be equally competent at recalling specific items of information, but with practice and experience experts "chunk" these items in sequences that relate to the goals of problem solution and use this capacity for further action and learning. This progression of developing competence from declarative knowledge to well-tuned functional knowledge is specifically described in John Anderson's well-known learning theory.

The theory has guided the development of instructional programs that have proven successful in several domains where the learning objective is the acquisition of efficient and functional cognitive skill. Computer-based instructional programs have been designed for learning problem solving in algebra, generating proofs in geometry, and teaching computer programming.¹¹⁴ These programs are unique in their reliance on an explicit learning theory, in the evaluation of their use in high school and university settings, and as a stage for systematically testing hypotheses about mechanisms of learning.¹¹⁵

A second area of study influencing conceptions of learning and instruction is cognitive science's increasing understanding of metacognitive processes and self-regulatory capabilities. Studies of the knowledge and skill of experts and cognitive development in children reveal the role that self-regulatory or control strategies play in competent performance. These regulatory activities enable the self-monitoring and executive control of one's performance.¹¹⁶ They include such strategies as predicting outcomes, planning ahead, apportioning one's time, explaining to one's self to improve understanding, noting failures to comprehend, and activating background knowledge. Individuals use such monitoring skills and evaluate the utility of strategies as they employ them; they also change strategies as required in the course of solving a problem or attempting to comprehend a situation. Although good learners have learned to use such skills, other individuals need to be taught to exercise these capabilities.

Instructional programs in reading, writing, and mathematics designed to foster the development of self-regulatory skills are a major area of research.¹¹⁷ The program for reading comprehension developed

by A. L. Brown and A. S. Palinscar has received sustained analysis, evaluation, and wide use.¹¹⁸ Students in this program acquire specific content knowledge and also learn a set of strategies for independently comprehending text material. The instructional procedure involves three major components: (1) instruction and practice with strategies that enable students to monitor their understanding of text; (2) provision, initially by a teacher, of an expert model of self-regulatory performance; and (3) a social setting that encourages joint observation and shared responsibility for learning.

A third area of influence on instructional design is the study of cultural experience and community participation. Research in cognitively oriented anthropology on cultural practices has brought attention to the high levels of performance that result from the demands of problem solving in everyday life in a community. Outside of formal schooling, individuals develop competence in solving verbal and quantitative problems that arise in community participation and in specialized work in trade and crafts.¹¹⁹ Participation in social practice is a fundamental form of learning relevant to learning theory and to the social settings of formal instructional environments. In this context, learning engages resources in the practices of the community.¹²⁰

Environments for instruction influenced by this work have been developed where students are involved in building and using knowledge for meaningful learning. The designers of these environments refer to L. S. Vygotsky's notion of creating a zone of proximal development where learners perform within their range of competence while being supported to realize potential levels of higher performance. In classroom learning communities or "communities for knowledge building," students participate in the transmission of knowledge by seeking, sharing, and acquiring knowledge among themselves with continued teacher guidance.¹²¹ These communities of knowledge building are distinguished by efforts to turn over processes that are usually under the teacher's control to the students. Students are helped to formulate goals, direct their own inquiry, monitor their understanding, and use the resources available to design their own settings for acquiring knowledge. In this participatory environment for learning, teachers and students share the expertise they have or take responsibility for finding out about needed knowledge that they can bring back to the group. Teachers often teach in response to student needs, rather than in a fixed sequence, but

the curriculum consists of topics to which students return for deepening knowledge and understanding. A community of discourse exists in which learning through constructive discussion, conjecture, questioning, criticism, and presenting evidence is practiced as the normal thing to do instead of the exception.¹²²

Conclusion

The programs have been empirically evaluated, including the assessment of student achievement and other learning outcomes; changes in teachers' concepts; and analysis of the fidelity of the educational environment to the designers' interpretation of theory. The coordination of practice and theory encouraged by Anderson, Reder, and Simon is proceeding in other innovations in educational practice. The combination of modern knowledge of learning and cognition and information about the outcomes and practices of the U.S. education systems should contribute to the improvement of student performance.

Despite the later interpretations of J. Dewey's philosophy, in his presidential address in 1899 before the American Psychological Association, he expressed the importance of developing a generative "linking science" between psychological theory and practical work just like the one in scientific medicine between natural science and the physician. Dewey said:

The real essence of the problem is found in . . . [a] connection between . . . the theorist and the practical worker—through the medium of the linking science. . . . It is the participation by the practical man in the theory . . . that determines . . . the effectiveness of the work done, and the moral freedom and personal development of the one engaged in it.¹²³

Without this, he said, educators are compelled to resort to purely arbitrary measures, to fall back upon mere routine traditions of school teaching, or to fly to the latest fad of pedagogical theorists—the latest panacea peddled out in school journals or teachers' institutes—just as the old physician relied upon his magic formula.¹²⁴

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The Use and Misuse of Research in Educational Reform

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MOST EDUCATORS and lawmakers still believe that good research produces good policy and that good policy produces good education in the schools. I suspect even jaundiced observers hold to this idealization on some level, since when things go wrong with school reform, when test scores go down or parents and teachers rebel against some newfangled idea, either policy or research is usually singled out as the culprit. This paper analyzes research and policy as well as the problems that occur when research and policy are joined to promote educational reform. Two recent state-initiated reforms illustrate how research may be distorted as it is absorbed by the educational system. Such distortions arise from structural flaws in educational governance. Research on practice enters the educational system at bureaucratic levels far removed from the schools and classrooms where practice takes place. This alienation of research from practice negatively affects education policy, compelling a reconsideration of how educational research, policy, and practice are connected in the current system.

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