

# Associative Learning

What's old is new again

---

*Anthony M. Harrison, U.S. Naval Research Laboratory*



NAVY CENTER FOR  
APPLIED RESEARCH IN  
ARTIFICIAL INTELLIGENCE



# Associative Learning

---





# Associative Learning

---

- Cross-cutting concern





# Associative Learning

---

- Cross-cutting concern
  - Embodiment





# Associative Learning

---

- Cross-cutting concern
  - Embodiment
  - Top/down perception





# Associative Learning

---

- Cross-cutting concern
  - Embodiment
  - Top/down perception
  - Long-term learning





# Associative Learning

---

- Cross-cutting concern
  - Embodiment
  - Top/down perception
  - Long-term learning
  - Memory for goals





# Associative Learning

---

- Cross-cutting concern
  - Embodiment
  - Top/down perception
  - Long-term learning
  - Memory for goals
  - Humanoid fire-fighter





# Associative Learning

---

- Cross-cutting concern
  - Embodiment
  - Top/down perception
  - Long-term learning
  - Memory for goals
  - Humanoid fire-fighter
  - IED bomb dogs





# Associative Learning

---

- Cross-cutting concern
  - Embodiment
  - Top/down perception
  - Long-term learning
  - Memory for goals
  - Humanoid fire-fighter
  - IED bomb dogs
- **Context is king!**





# Associative Learning

---

- Cross-cutting concern
  - Embodiment
  - Top/down perception
  - Long-term learning
  - Memory for goals
  - Humanoid fire-fighter
  - IED bomb dogs
- **Context is king!**
  - **Yet it can't be learned**





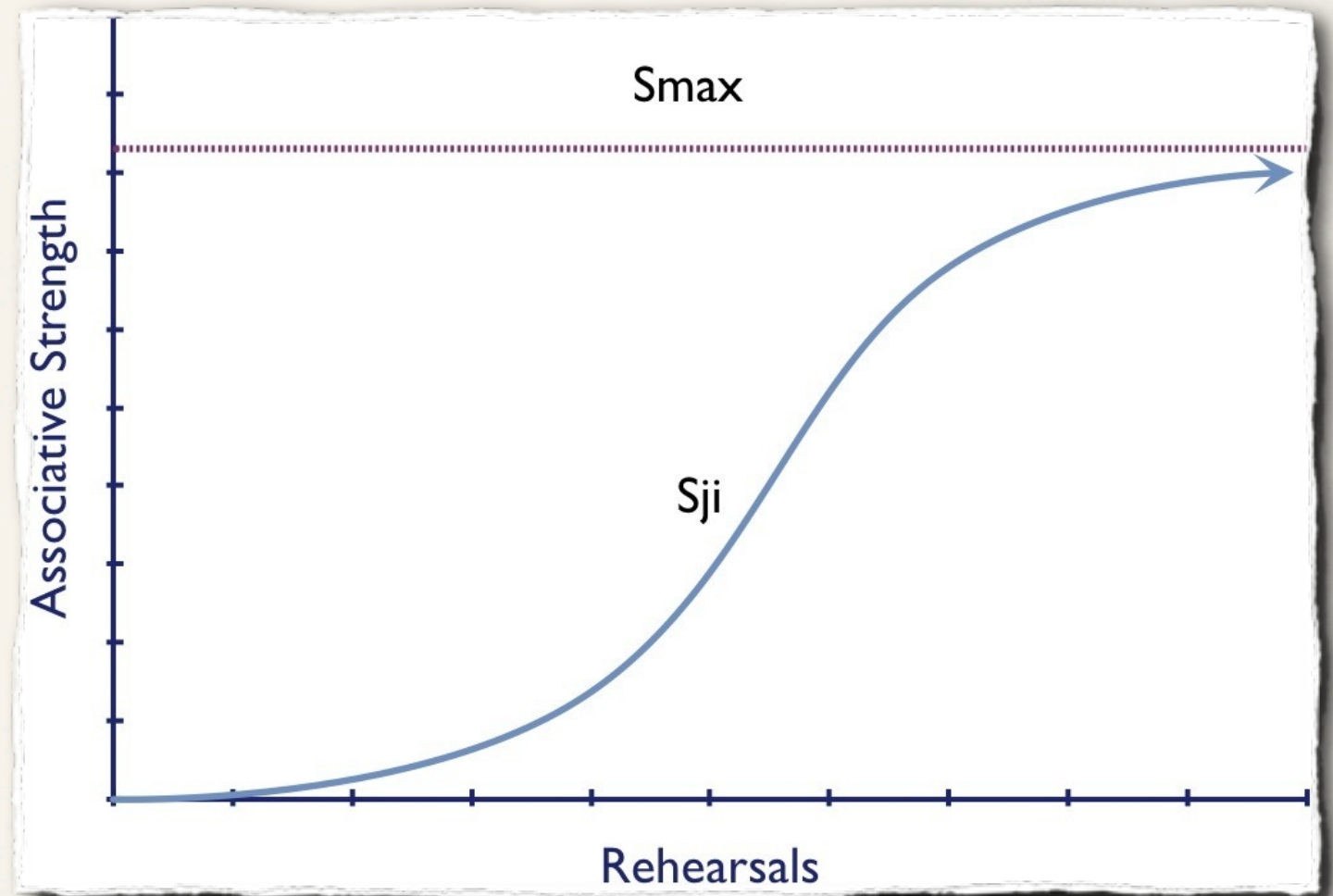
# What do we need?

---



# What do we need?

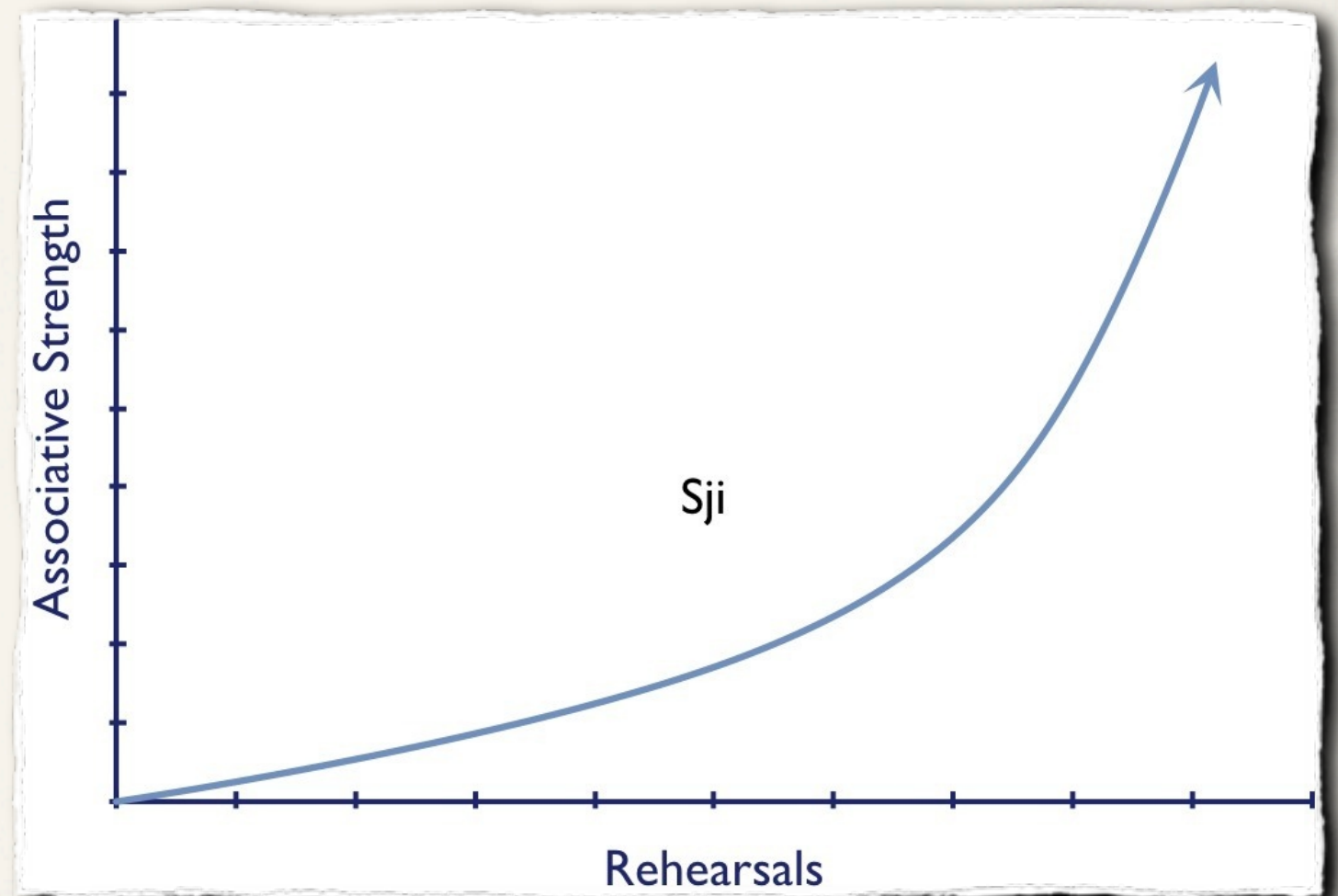
- Associations should be defined by model's behavior





# What do we need?

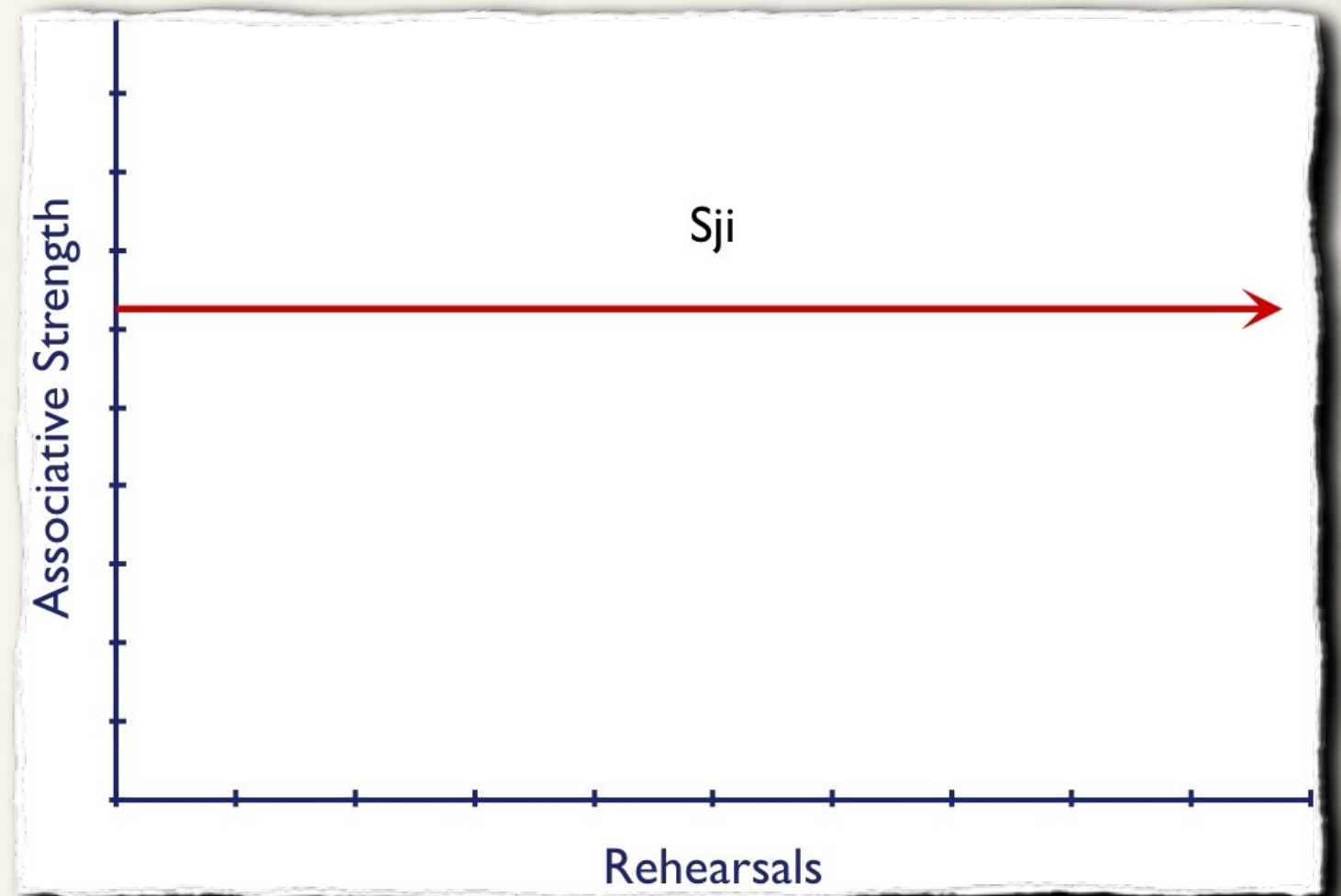
- Associations should be defined by model's behavior
- Associations should strengthen with exposure





# What do we need?

- Associations should be defined by model's behavior
- Associations should strengthen with exposure
- Not be **defined** solely by **symbolic structure**





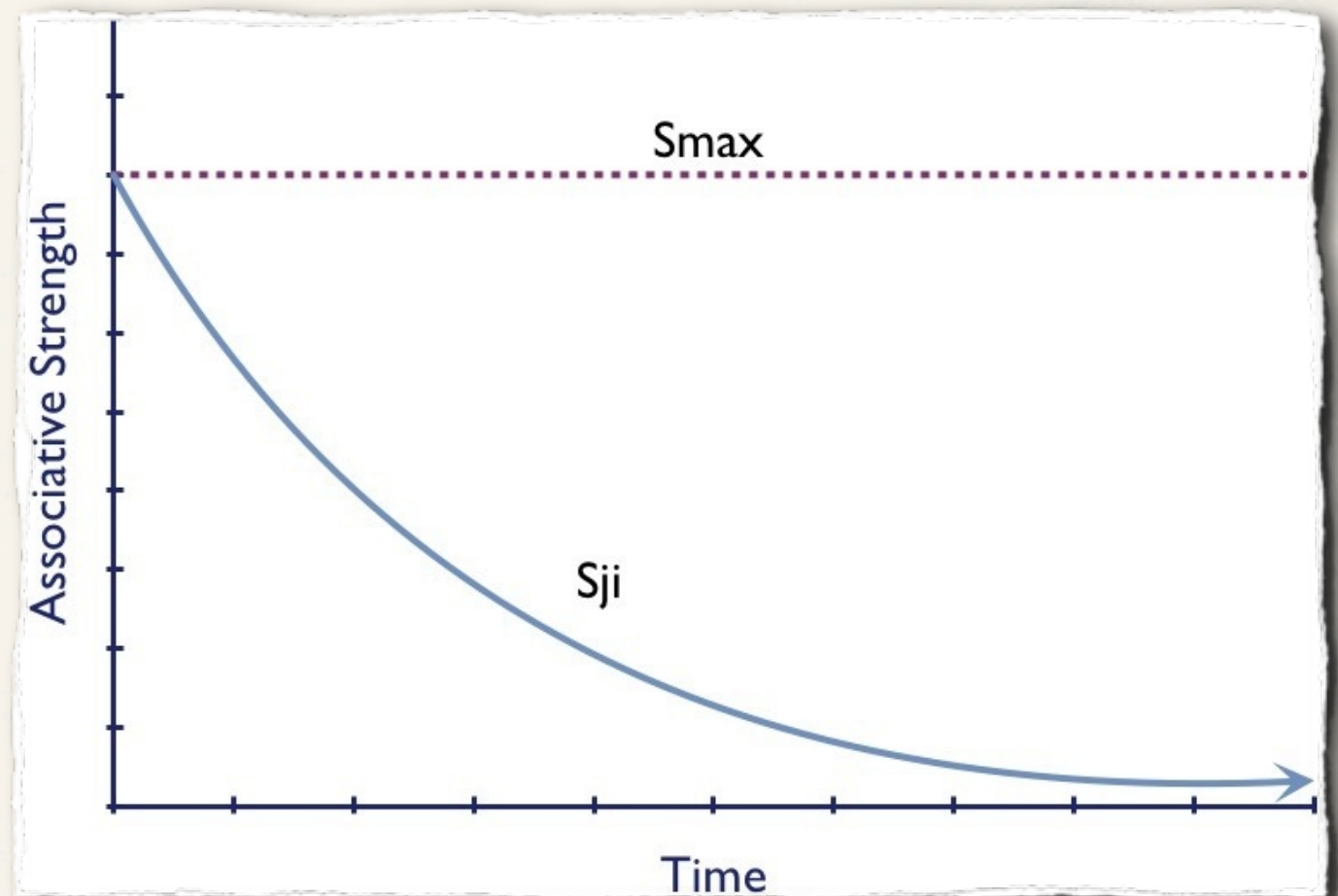
# What do we need?

---



# What do we need?

- Associations should be stable or decay with time

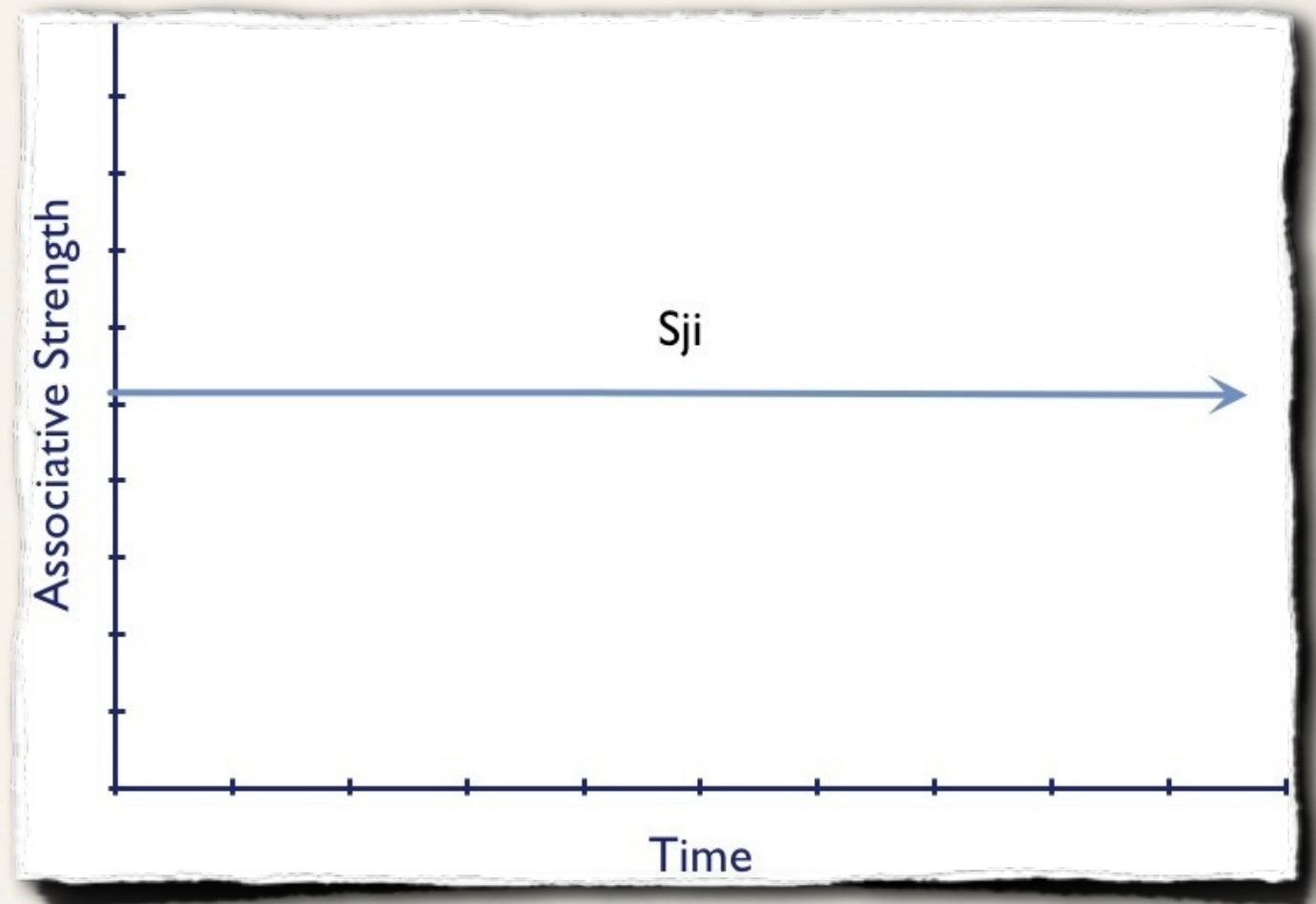




# What do we need?

---

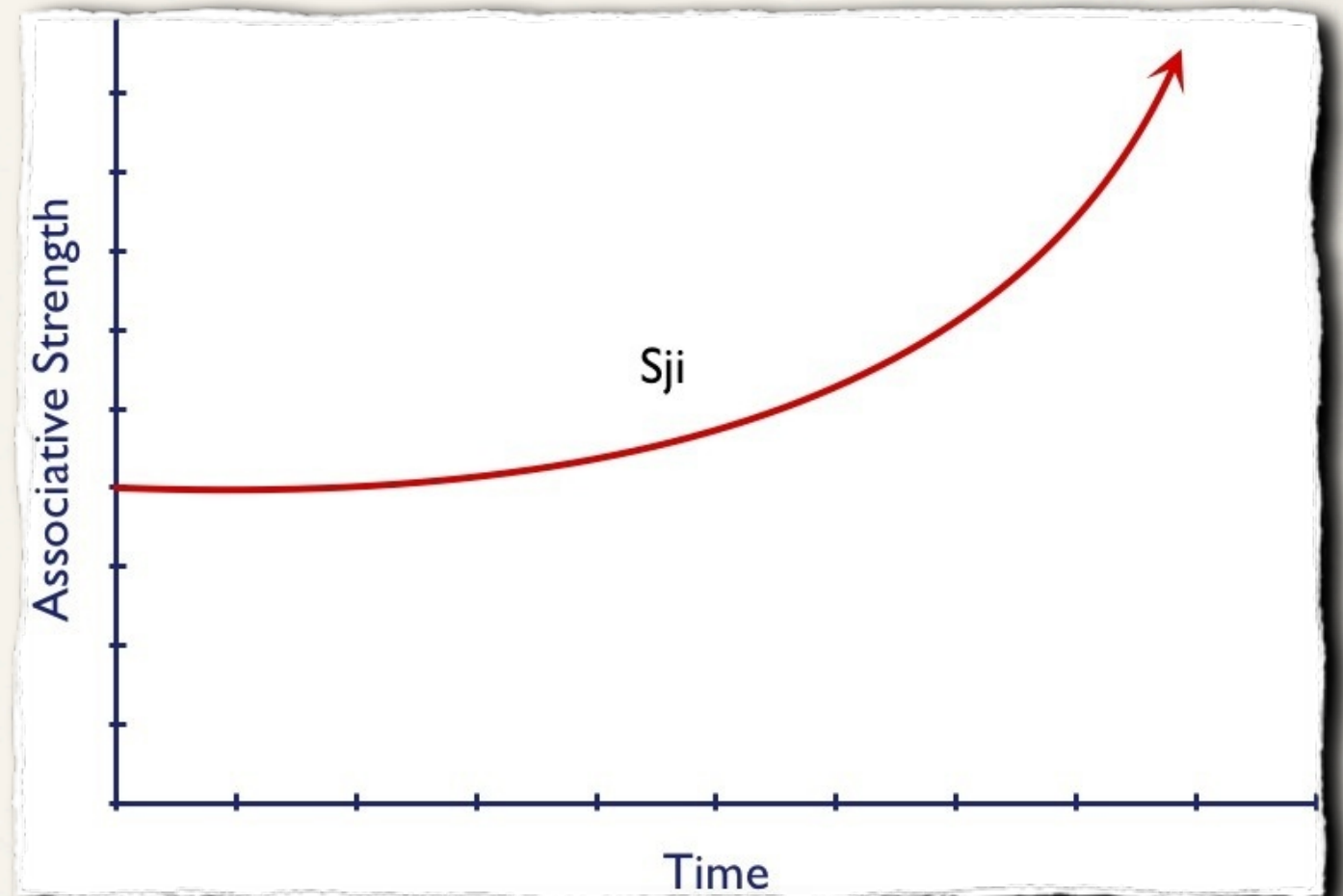
- Associations should be stable or decay with time
- All other things being equal





# What do we need?

- Associations should be stable or decay with time
- All other things being equal
- Never **strengthen sans rehearsal**





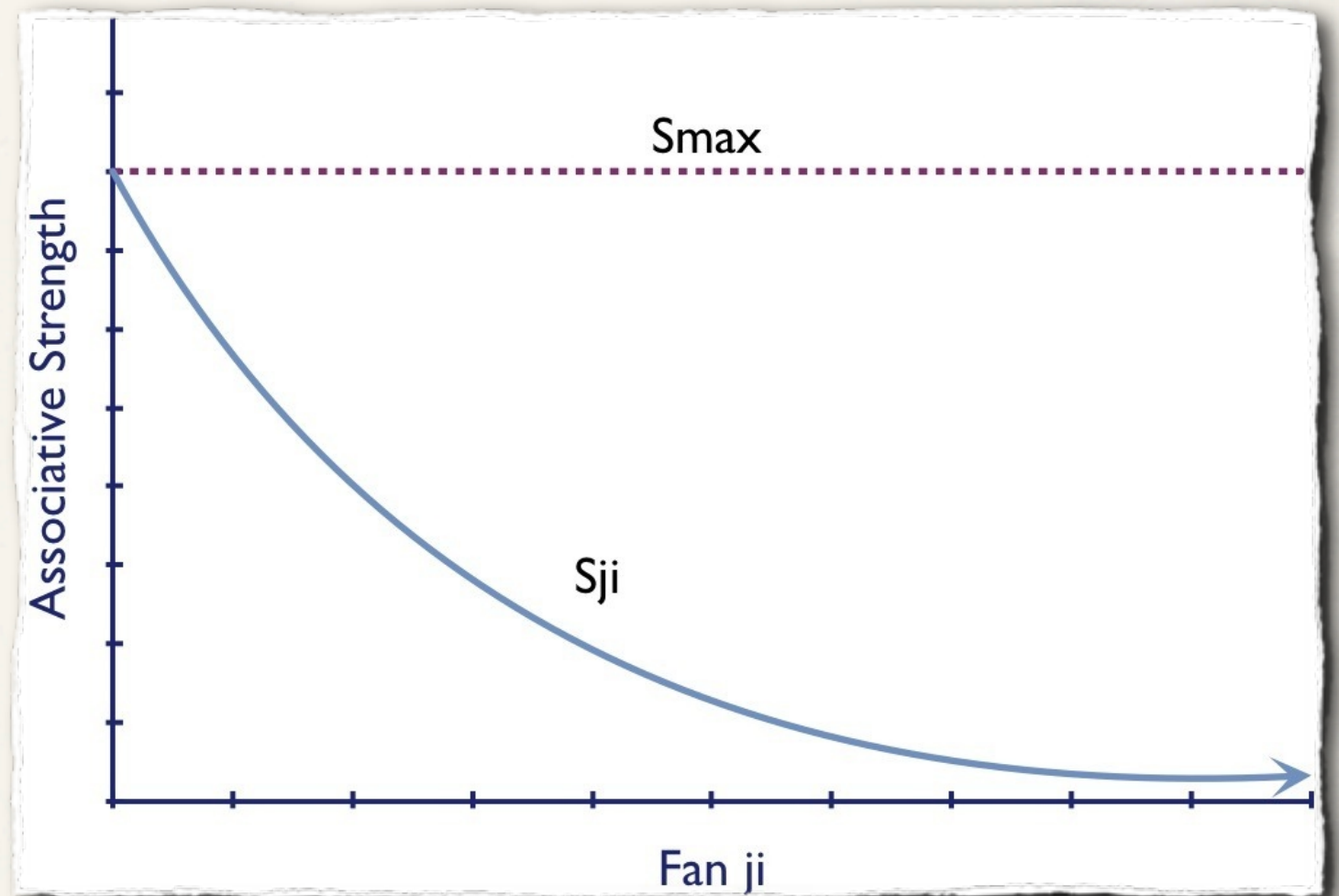
# What do we need?

---



# What do we need?

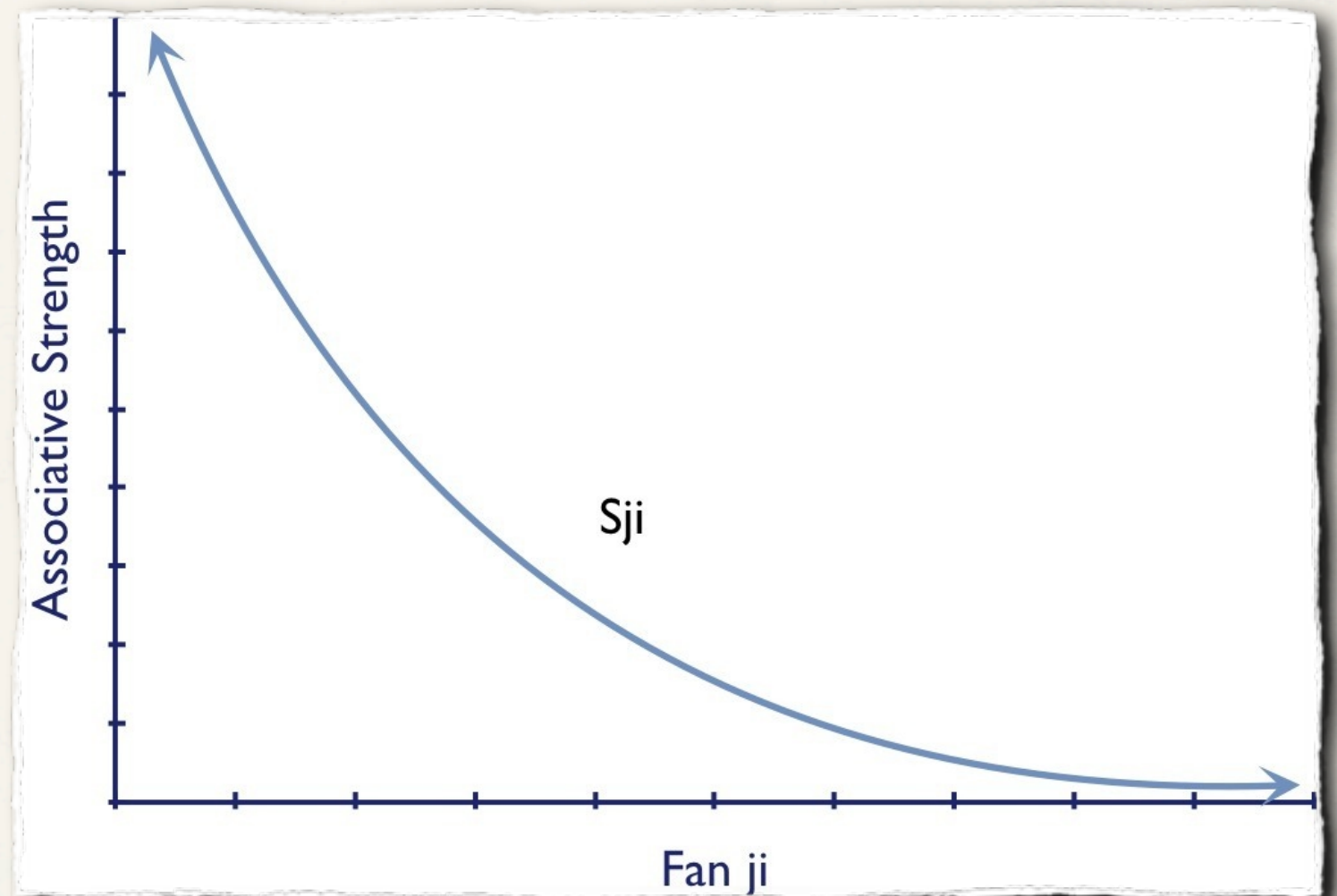
- Associations that weaken with fanji





# What do we need?

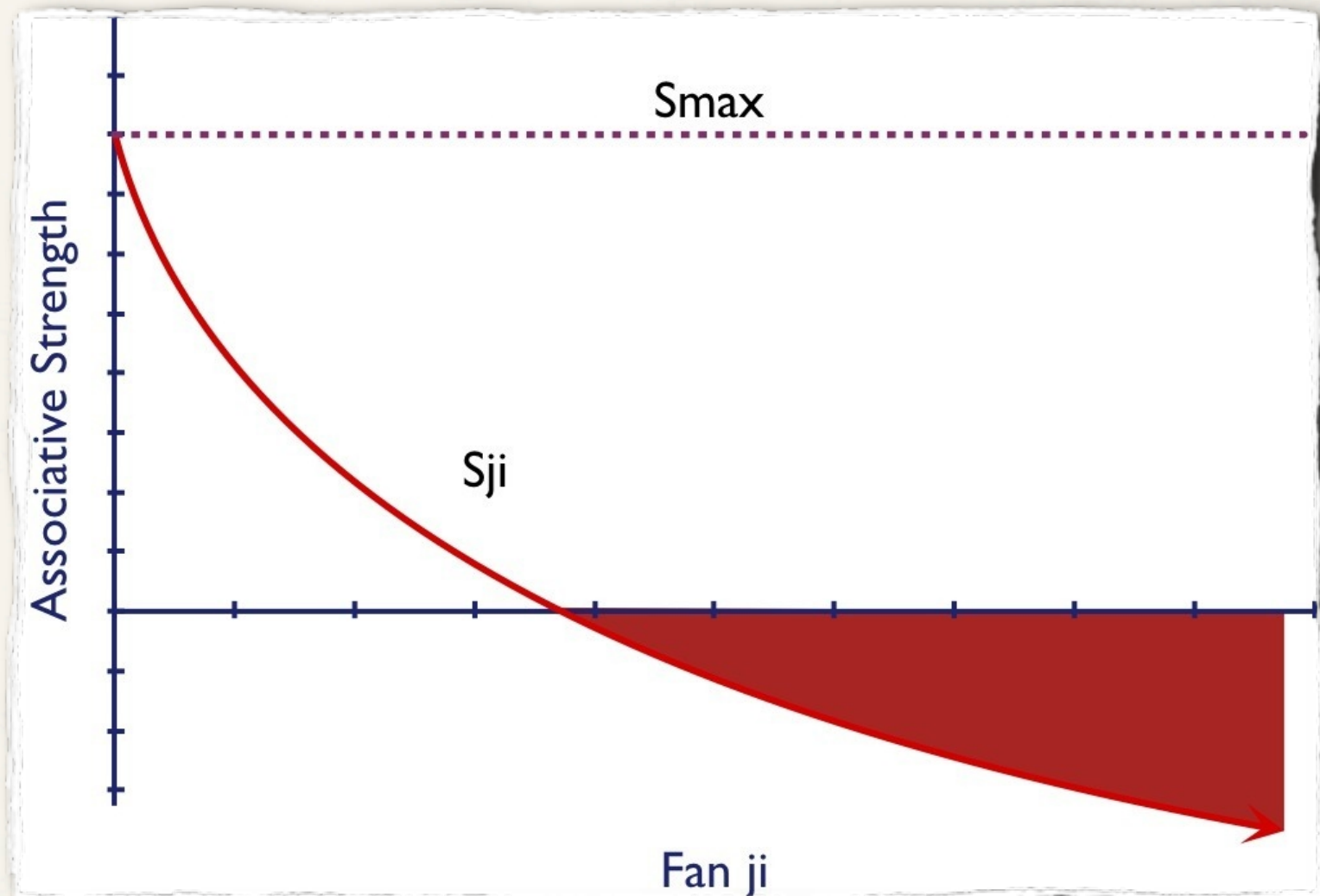
- Associations that weaken with fanji





# What do we need?

- Associations that weaken with fanji
- Never be **inhibitory** (unless desired)



# ACT-R 6.0

---



# ACT-R 6.0

---

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

# ACT-R 6.0

---

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

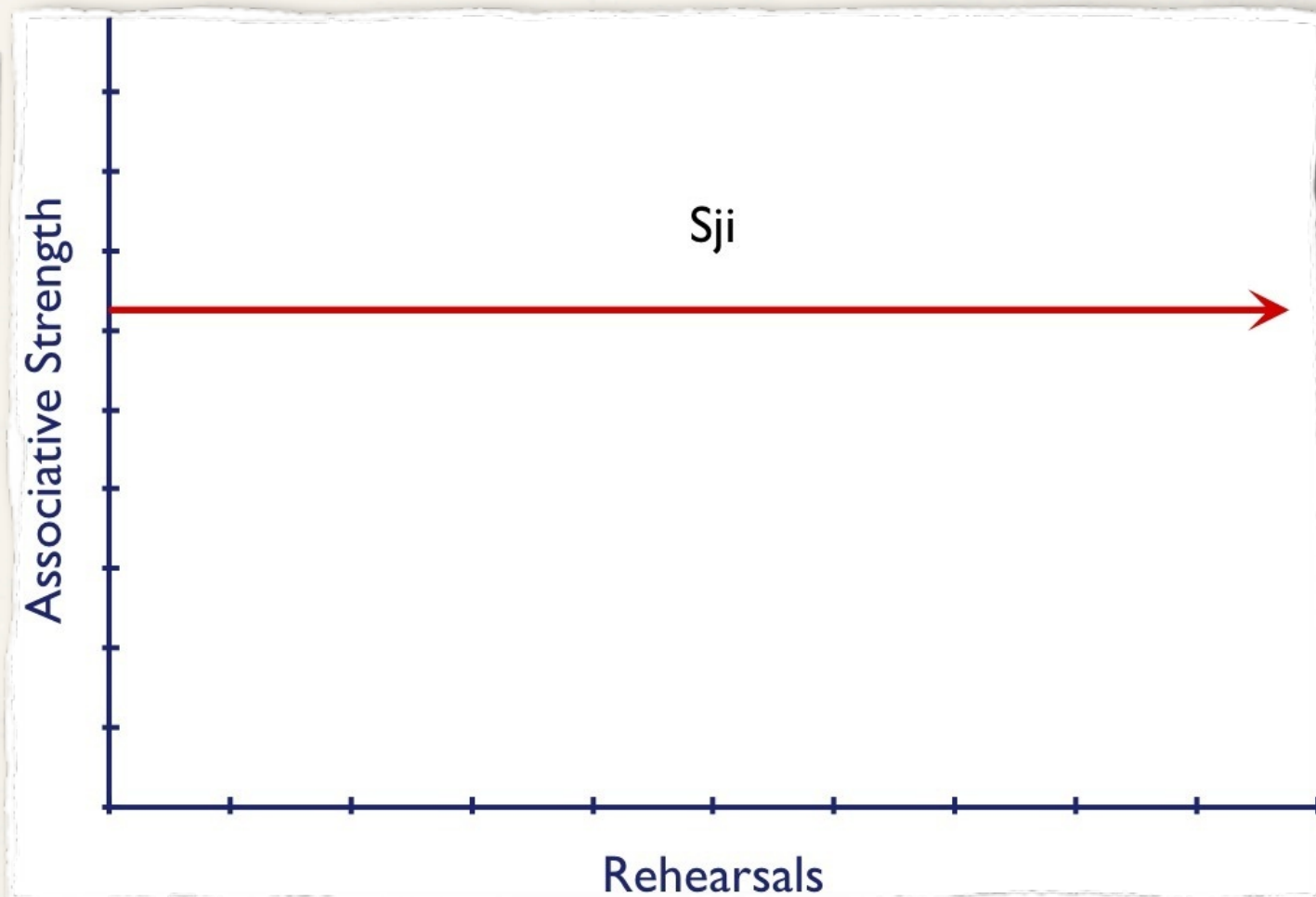
$$S_{ji} = S_{max} - \ln(fan_{ji})$$



# ACT-R 6.0

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$$S_{ji} = S_{max} - \ln(fan_{ji})$$

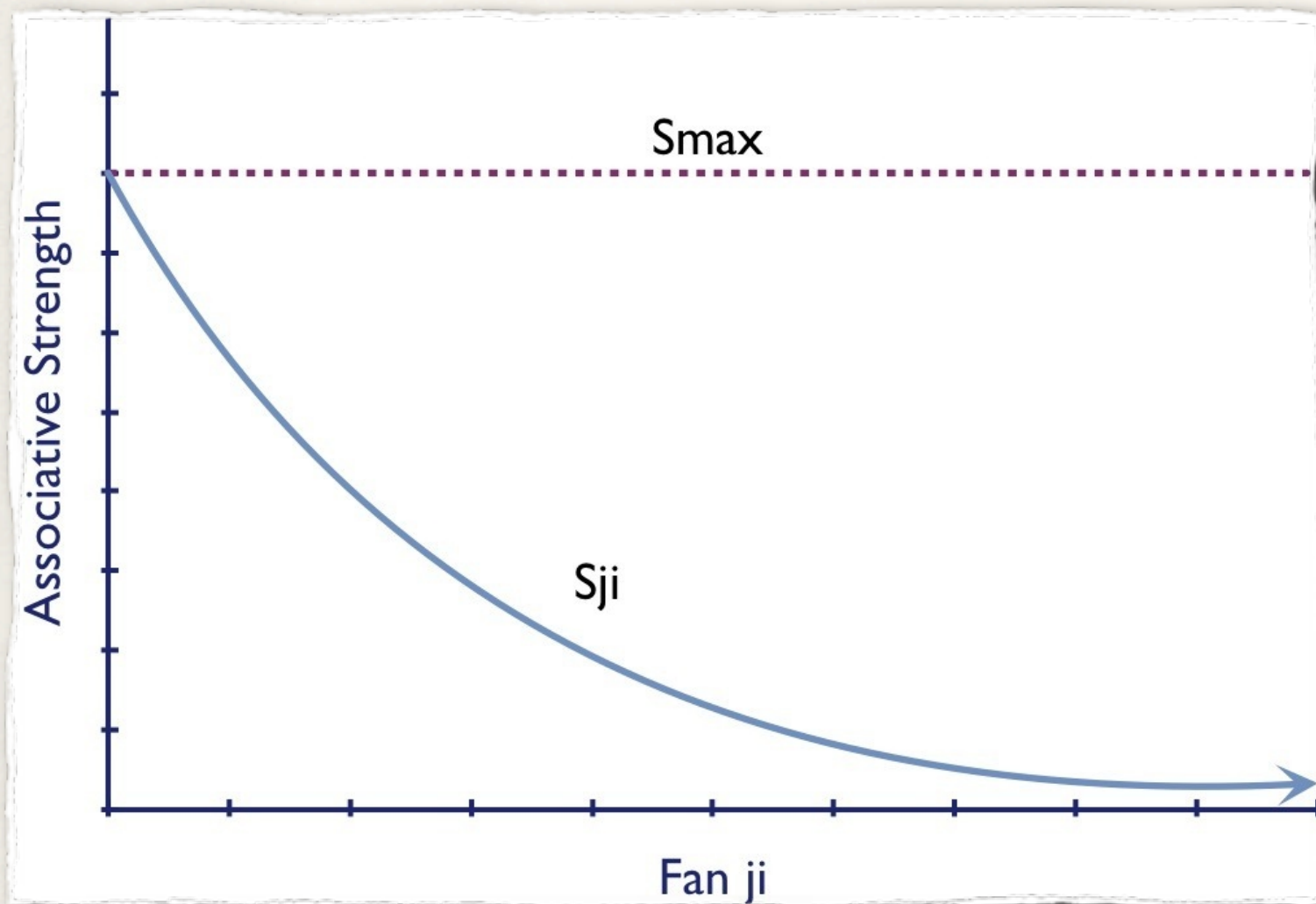


# ACT-R 6.0

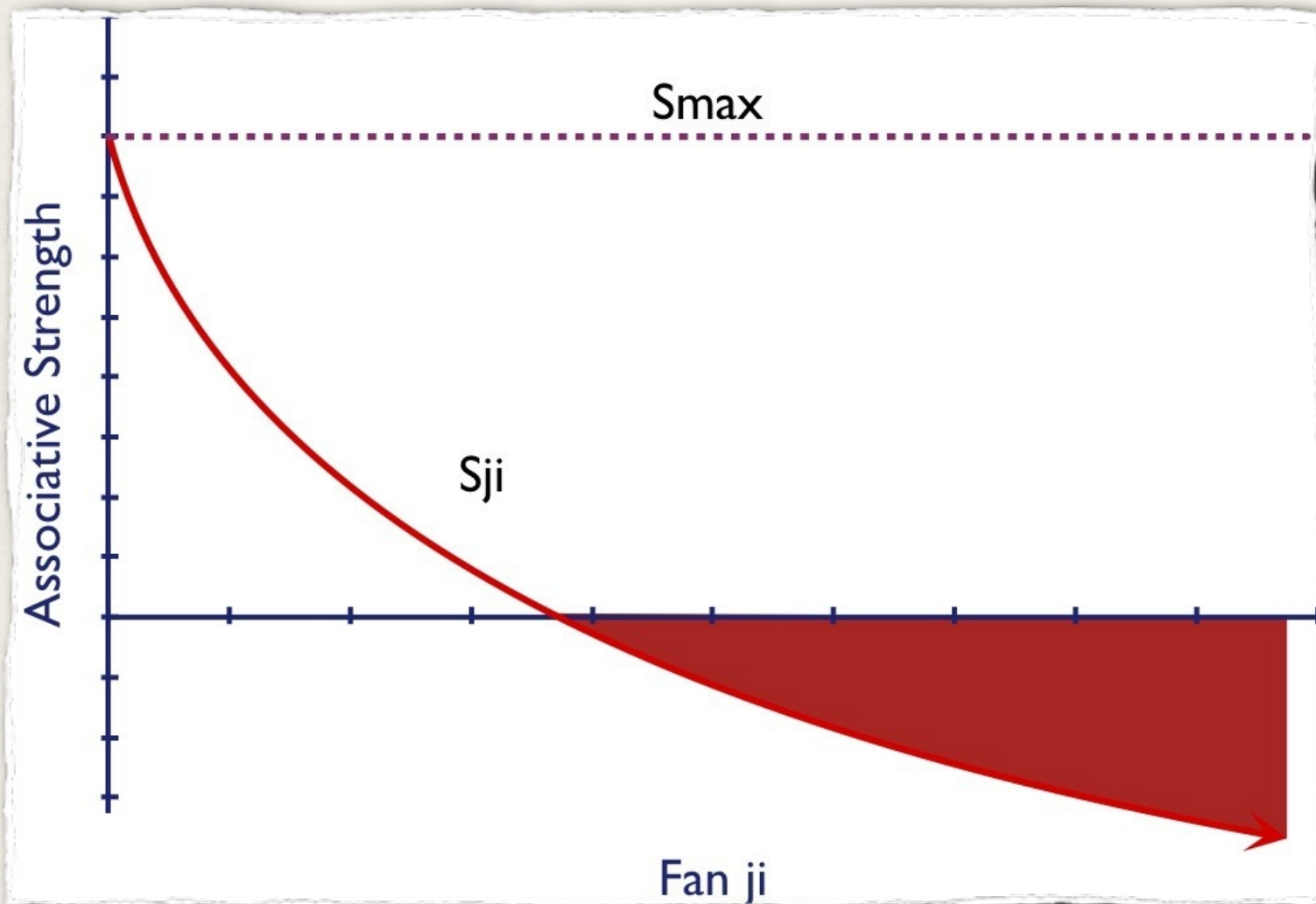
---



# ACT-R 6.0



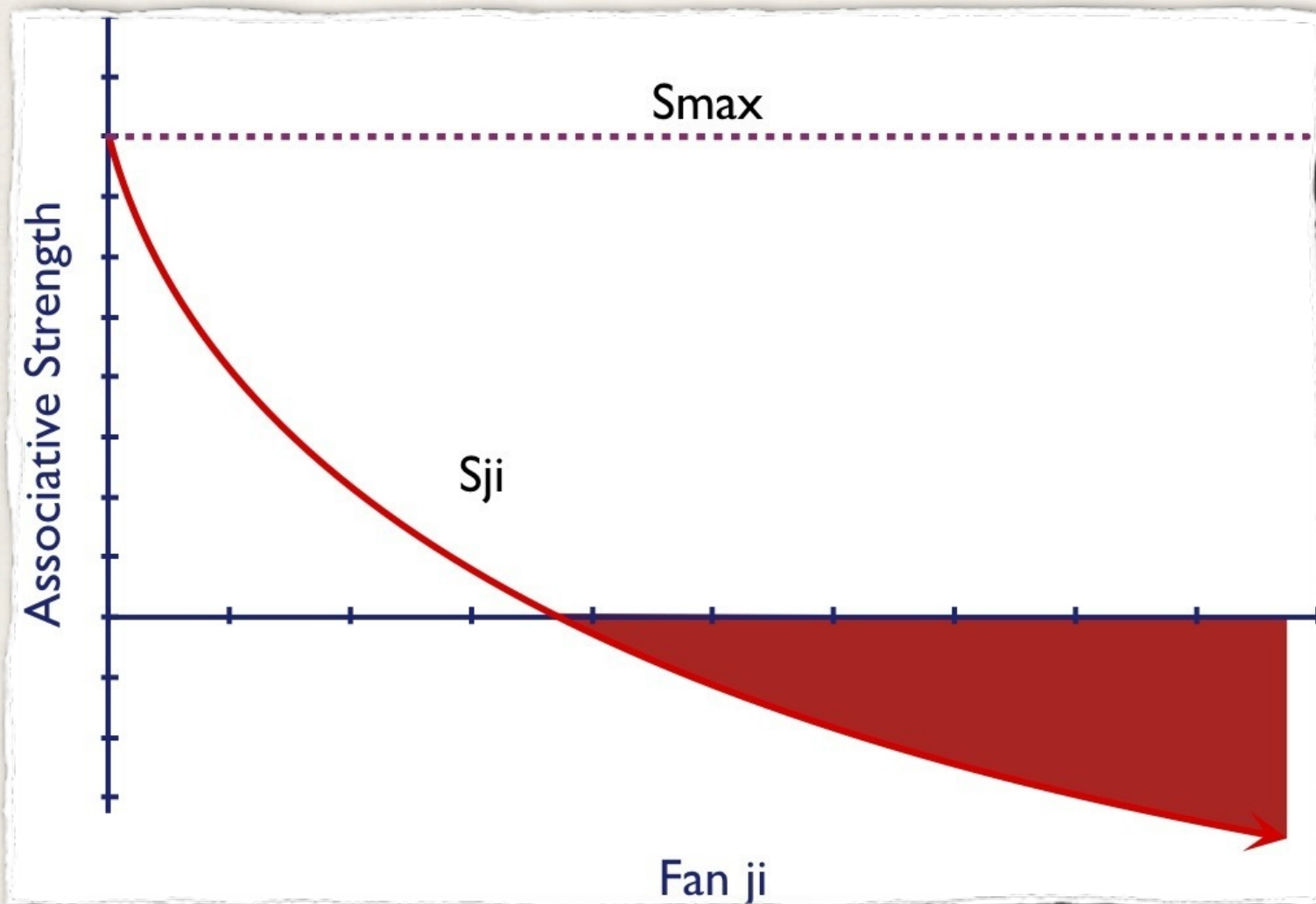
# ACT-R 6.0



- Fanji growth is unbounded

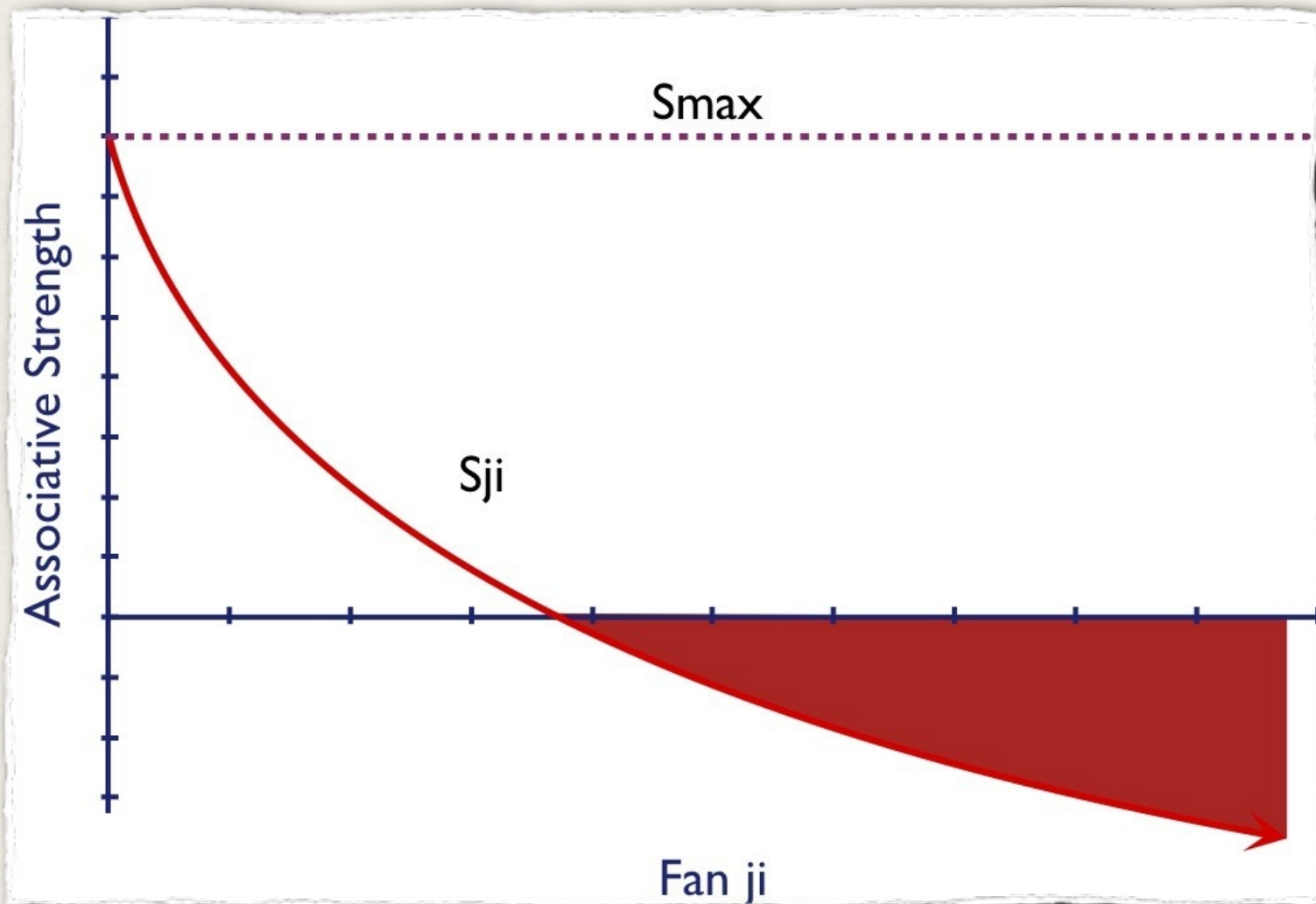


# ACT-R 6.0



- Fanji growth is unbounded
- Fan can grow quite large for some chunks

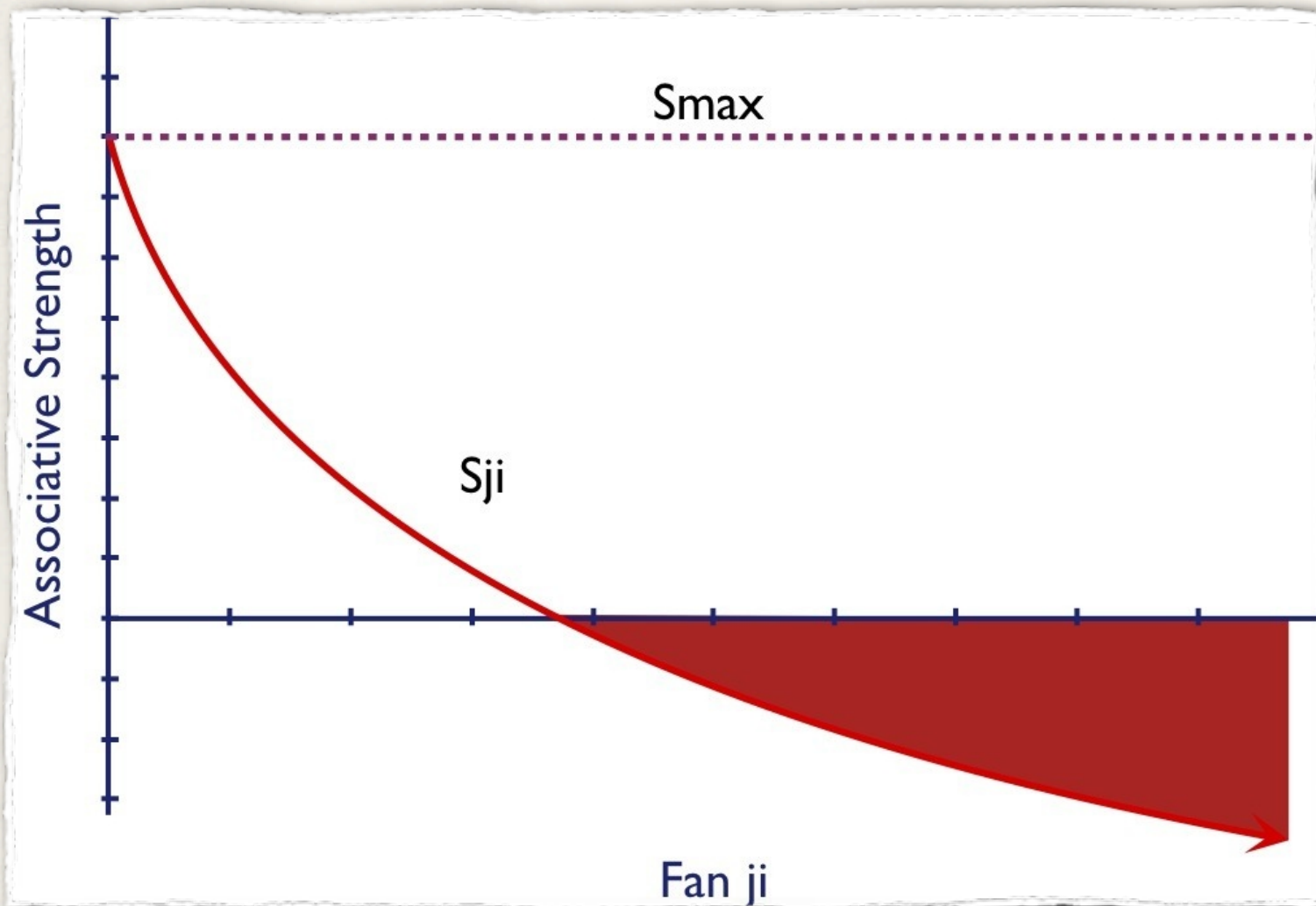
# ACT-R 6.0



- Fanji growth is unbounded
- Fan can grow quite large for some chunks
  - ✧ States

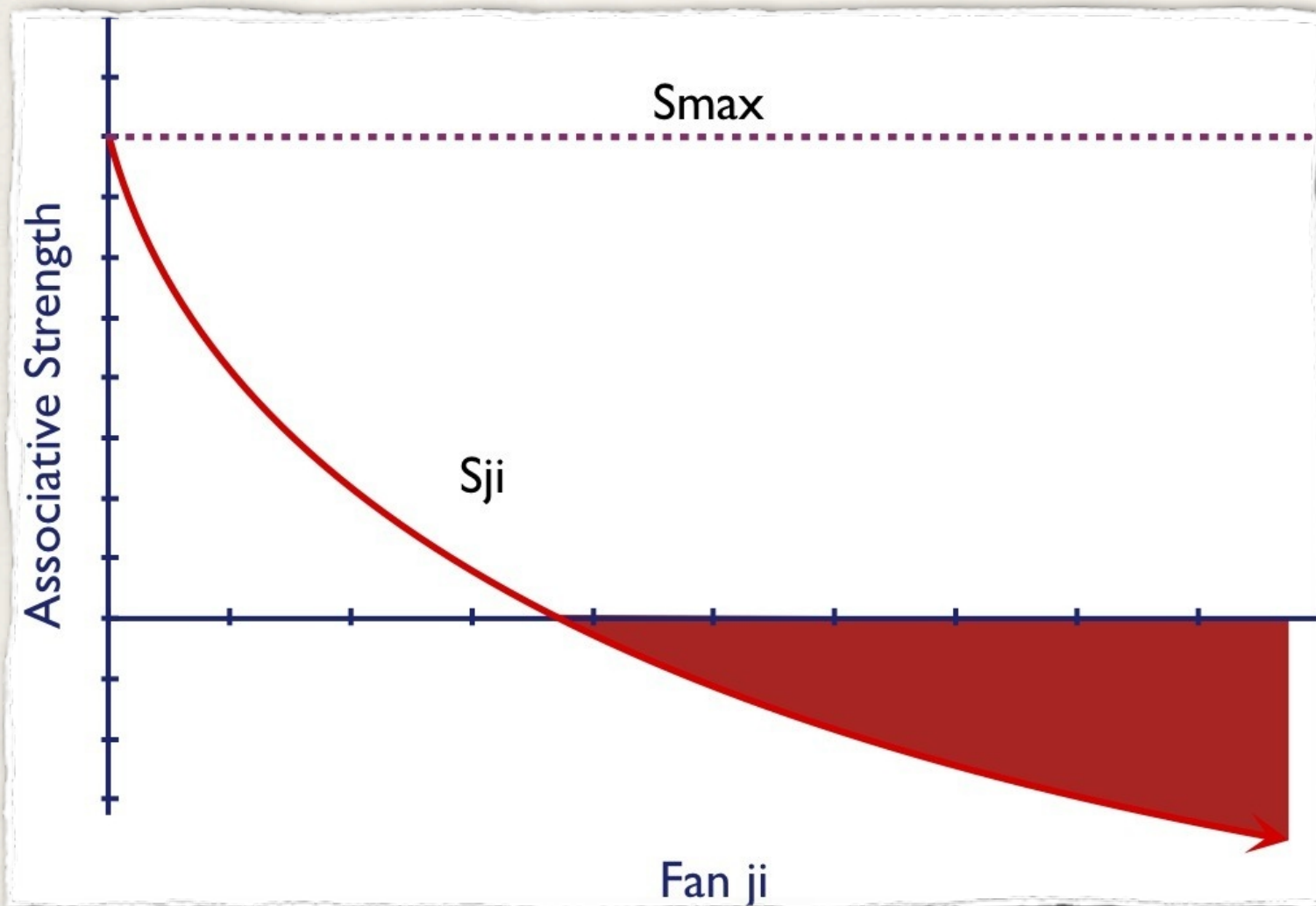


# ACT-R 6.0



- Fanji growth is unbounded
- Fan can grow quite large for some chunks
  - ✧ States
  - ✧ Visual properties

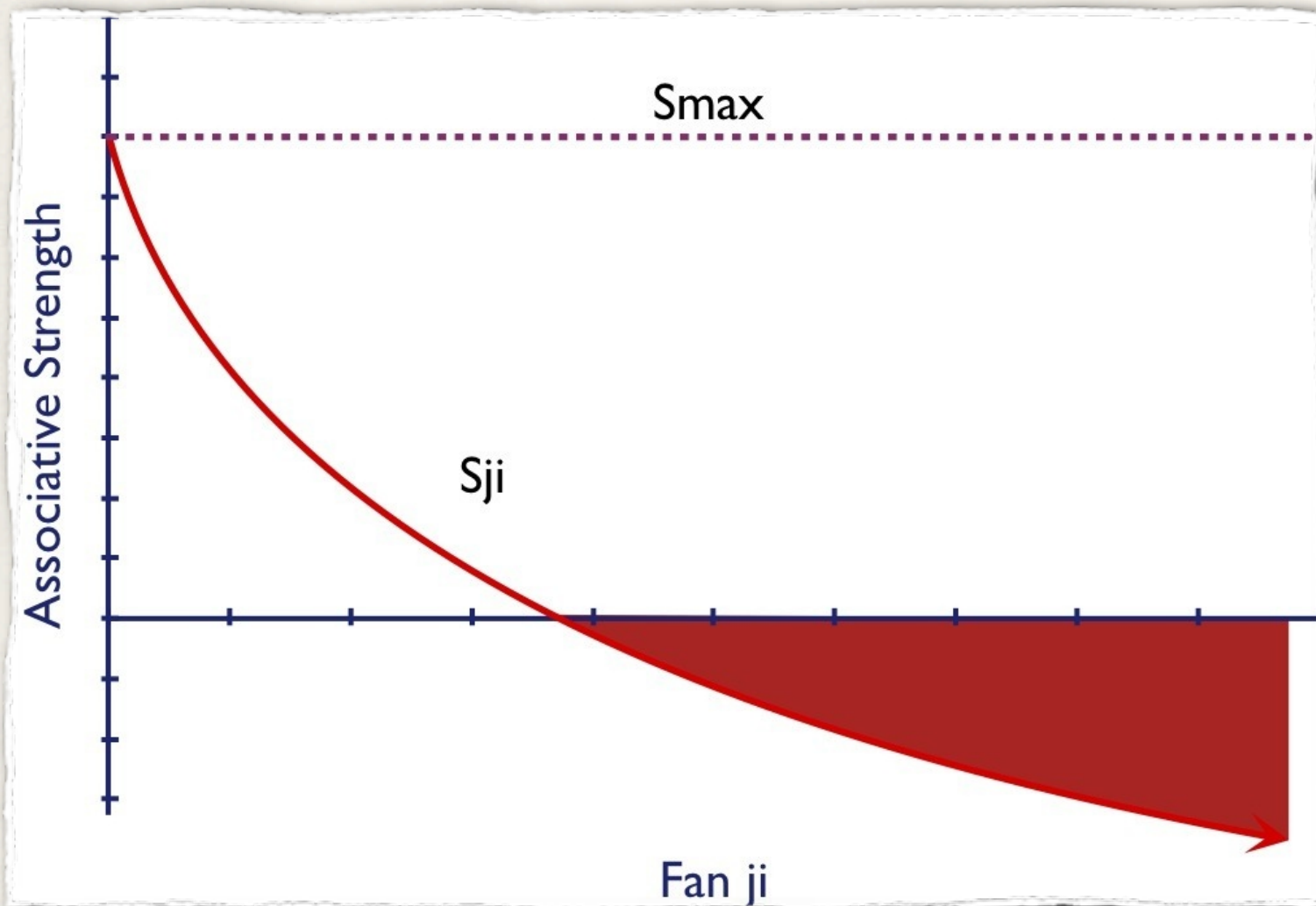
# ACT-R 6.0



- Fanji growth is unbounded
- Fan can grow quite large for some chunks
  - \* States
  - \* Visual properties
- $S_{ji}$  quickly becomes inhibitory

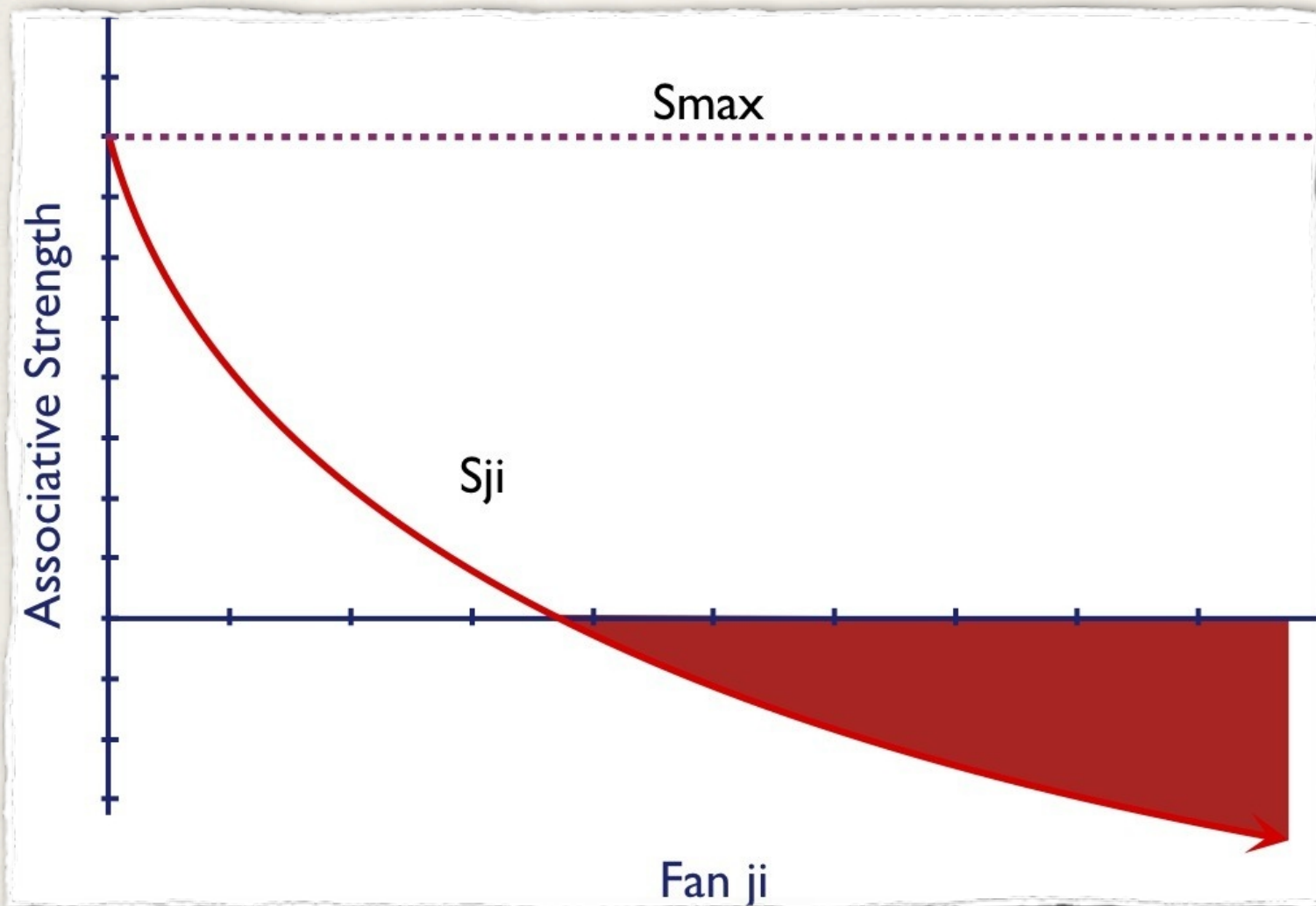


# ACT-R 6.0



- Fanji growth is unbounded
- Fan can grow quite large for some chunks
  - \* States
  - \* Visual properties
- $S_{ji}$  quickly becomes inhibitory
- Can be **catastrophic**

# ACT-R 6.0



- Fanji growth is unbounded
- Fan can grow quite large for some chunks
  - ✧ States
  - ✧ Visual properties
- $S_{ji}$  quickly becomes inhibitory
- Can be **catastrophic**
- $S_{max}$  becomes a breaking point between models



# ACT-R 4

---

# ACT-R 4

---

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$



# ACT-R 4

---

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

(p sample  
=goal>  
isa goal  
slot1 chunkj1  
slot2 chunkj2  
=retrieval>  
isa something

# ACT-R 4

---

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$N_i$  : Chunk  $i$  was *needed*  
(retrieved)

(p sample  
=goal>  
isa goal  
slot1 chunkj1  
slot2 chunkj2  
*Needed* =retrieval>  
isa something



# ACT-R 4

---

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$N_i$  : Chunk  $i$  was *needed*  
(retrieved)

$C_j$  : Chunk  $j$  in the *context*  
(slot value of goal)

(p sample  
=goal>  
isa goal

*Context* slot1 chunkj1  
slot2 chunkj2

*Needed* =retrieval>  
isa something

# ACT-R 4

---



# ACT-R 4

---

$$\frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

# ACT-R 4

---

$$\frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$$\frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)} = \frac{\frac{P(C_j N_i)}{P(N_i)}}{\frac{P(C_j \bar{N}_i)}{P(\bar{N}_i)}}$$



# ACT-R 4

---

$$\frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$$\frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)} = \frac{\frac{P(C_j N_i)}{P(N_i)}}{\frac{P(C_j \bar{N}_i)}{P(\bar{N}_i)}}$$

$$\frac{P(C_j N_i) P(\bar{N}_i)}{P(C_j \bar{N}_i) P(N_i)}$$

# ACT-R 4

---

$$\frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$$\frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)} = \frac{\frac{P(C_j N_i)}{P(N_i)}}{\frac{P(C_j \bar{N}_i)}{P(\bar{N}_i)}}$$

$$\frac{P(C_j N_i) P(\bar{N}_i)}{P(C_j \bar{N}_i) P(N_i)}$$

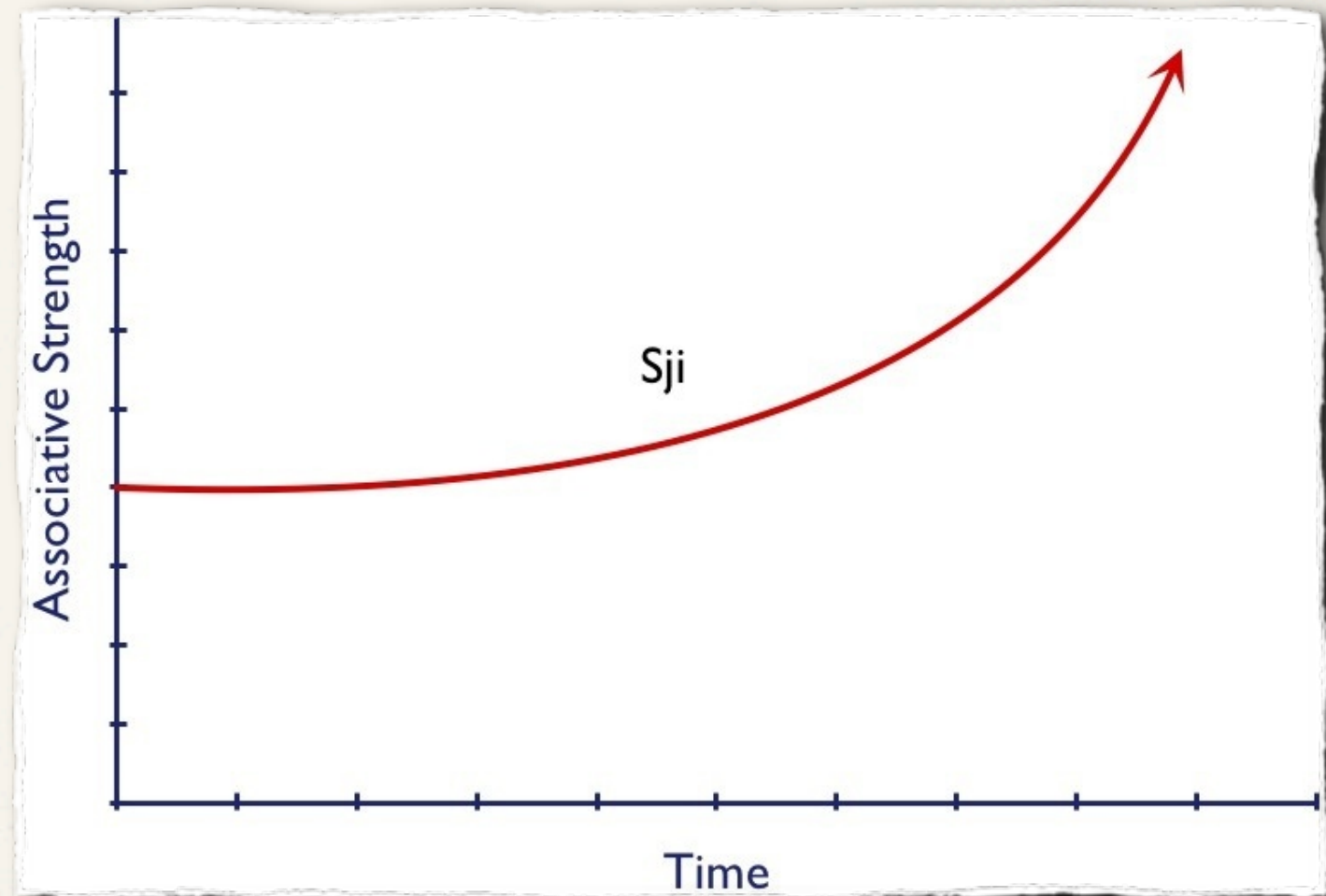


# ACT-R 4

$$\frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$$\frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)} = \frac{\frac{P(C_j N_i)}{P(N_i)}}{\frac{P(C_j \bar{N}_i)}{P(\bar{N}_i)}}$$

$$\frac{P(C_j N_i) P(\bar{N}_i)}{P(C_j \bar{N}_i) P(N_i)}$$



# Back to the Future

---



# Back to the Future

---

- Update 4.0 equations and mappings to match pattern 6.0

# Back to the Future

---

- Update 4.0 equations and mappings to match pattern 6.0
- Associations defined by production-level co-occurrence



# Back to the Future

---

- Update 4.0 equations and mappings to match pattern 6.0
- Associations defined by production-level co-occurrence
  - Subsumes containment associative links

# Back to the Future

---

- Update 4.0 equations and mappings to match pattern 6.0
- Associations defined by production-level co-occurrence
  - Subsumes containment associative links
- Have looked at associations across a single buffer (retrieval priming retrieval, ala Richard Young)



# Associative Proposal

---

# Associative Proposal

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$



# Associative Proposal

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

(p sample  
=goal>  
isa goal  
slot1 chunkj1  
slot2 chunkj2  
=retrieval>  
isa something  
slot1 chunkj3  
=visual>  
isa visual-object  
color red

==>

# Associative Proposal

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$N_i$  : Chunk  $i$  was *needed*  
(matched in **any** buffer)

(p sample

*Needed* =goal>

isa goal

slot1 chunkj1

slot2 chunkj2

*Needed* =retrieval>

isa something

slot1 chunkj3

*Needed* =visual>

isa visual-object

color red

==>



# Associative Proposal

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$N_i$  : Chunk  $i$  was *needed*  
(matched in **any** buffer)

$C_j$  : Chunk  $j$  in the *context*  
(slot value of any matched  
chunk in **any** buffer)

*Needed*

(p sample  
=goal>

isa goal

*Context*

slot1 chunkj1

slot2 chunkj2

*Needed*

=retrieval>

isa something

*Context*

slot1 chunkj3

*Needed*

=visual>

isa visual-object

*Context*

color red

==>

# Not Quite Bayesian

---

approx.



# Not Quite Bayesian

---

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

approx.

# Not Quite Bayesian

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$$S_{ji} = S_{max} - \ln(fan_{ji})$$

$$\frac{P(C_j|N_i) P(\bar{N}_i)}{P(C_j|\bar{N}_i) P(N_i)}$$

approx.



# Not Quite Bayesian

$$\frac{P(N_i|C)}{P(\bar{N}_i|C)} = \frac{P(N_i)}{P(\bar{N}_i)} \prod_j \frac{P(C_j|N_i)}{P(C_j|\bar{N}_i)}$$

$$S_{ji} = S_{max} - \ln(fan_{ji})$$

$$\frac{P(C_j|N_i) P(\bar{N}_i)}{P(C_j|\bar{N}_i) P(N_i)}$$

$$\frac{F(C_j|N_i)}{F(C_j|\bar{N}_i)}$$

approx.

$$\frac{1}{fan_{ji}}$$

# Alternatives

---



# Alternatives

---

$$\frac{F(C_j \ N_i)}{F(C_j)}$$

$$\frac{F(C_j \ N_i)}{1 + F(C_j \ \bar{N}_i)}$$

[0-1]

# Alternatives

---

[0-1]

$$\frac{F(C_j N_i)}{F(C_j)}$$

$$\frac{F(C_j N_i)}{1 + F(C_j \bar{N}_i)}$$

$$S_{ji} = S_{max}^{\frac{F(C_j N_i)}{F(C_j \bar{N}_i)}}$$



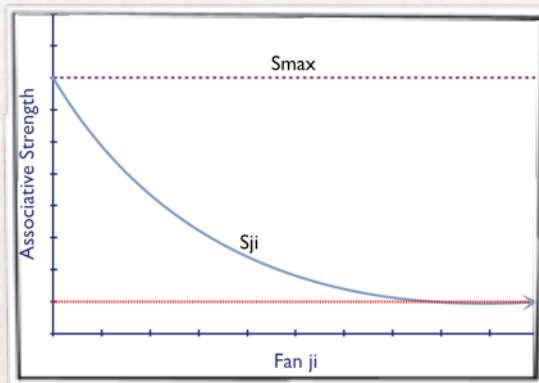
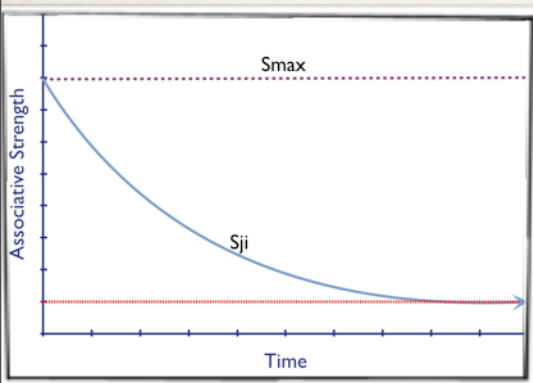
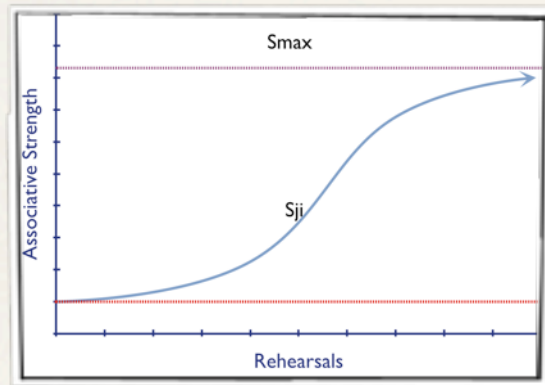
# Alternatives

[0-1]

$$\frac{F(C_j N_i)}{F(C_j)}$$

$$\frac{F(C_j N_i)}{1 + F(C_j \bar{N}_i)}$$

$$S_{ji} = S_{max} \frac{F(C_j N_i)}{F(C_j \bar{N}_i)}$$



# Alternatives

[0-1]

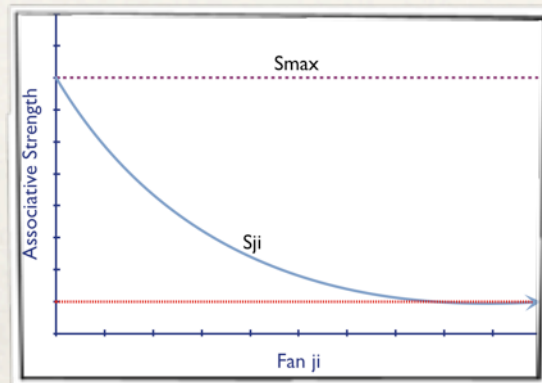
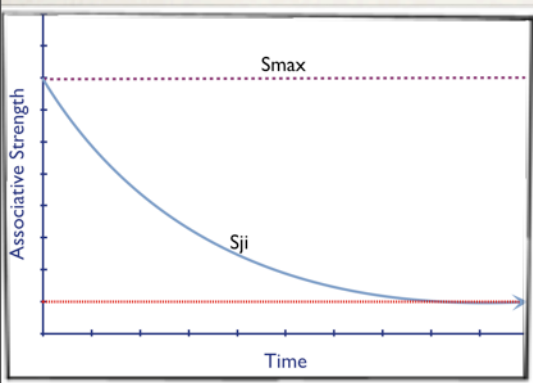
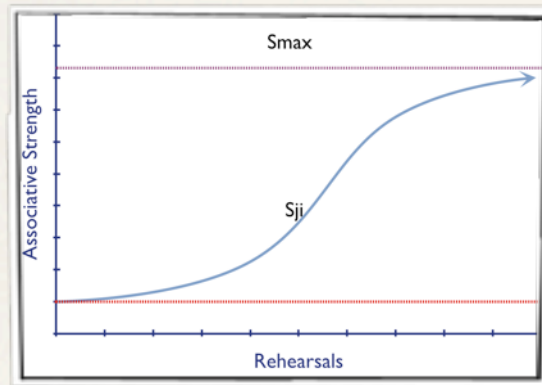
$$\frac{F(C_j N_i)}{F(C_j)}$$

$$\frac{F(C_j N_i)}{1 + F(C_j \bar{N}_i)}$$

$$\frac{F(C_j N_i)}{F(C_j \bar{N}_i)}$$

[0-Infinity]

$$S_{ji} = S_{max} \frac{F(C_j N_i)}{F(C_j \bar{N}_i)}$$





# Alternatives

[0-1]

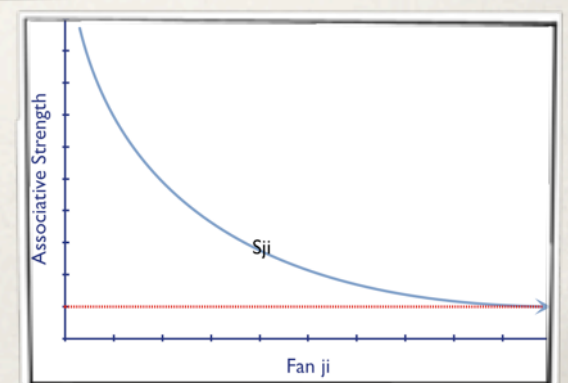
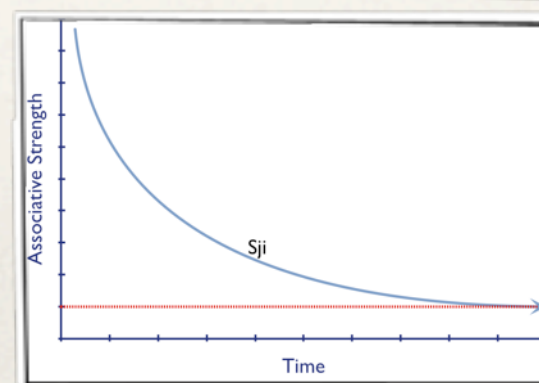
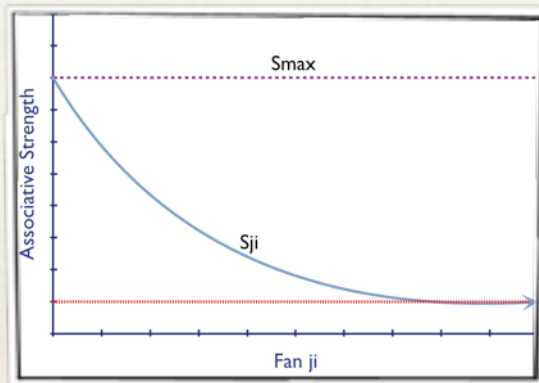
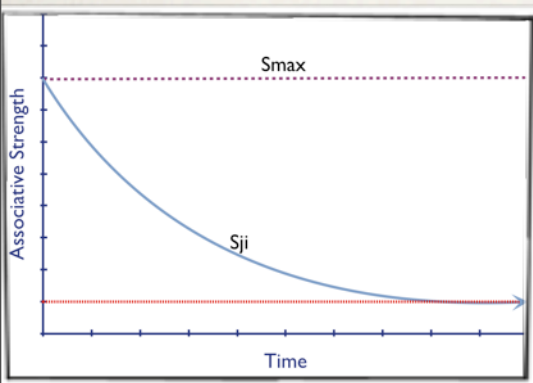
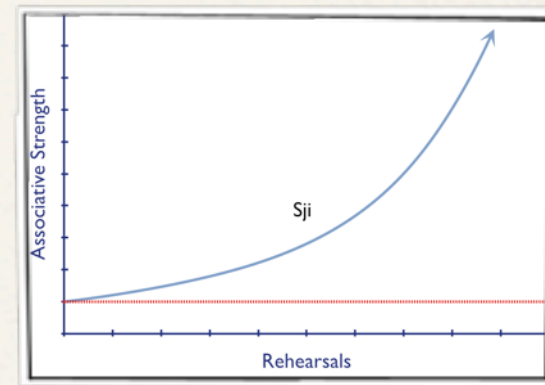
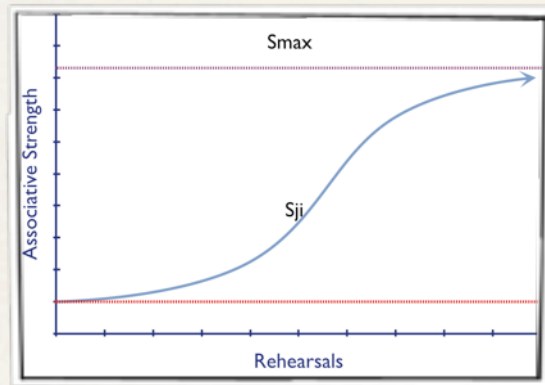
$$\frac{F(C_j N_i)}{F(C_j)}$$

$$\frac{F(C_j N_i)}{1 + F(C_j \bar{N}_i)}$$

$$\frac{F(C_j N_i)}{F(C_j \bar{N}_i)}$$

[0-Infinity]

$$S_{ji} = S_{max} \frac{F(C_j N_i)}{F(C_j \bar{N}_i)}$$



# Model Fit

---



# Model Fit

---

- ❖ Tucker & Ellis (1998) (Harrison & Trafton (2010) )

# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ✧ Latency  $R^2=0.94$ , RMSE=5.6ms



# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ✧ Latency  $R^2=0.94$ , RMSE=5.6ms
  - ✧ Accuracy  $R^2=0.82$ , RMSE=6%

# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ✧ Latency  $R^2=0.94$ , RMSE=5.6ms
  - ✧ Accuracy  $R^2=0.82$ , RMSE=6%



# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ✧ Latency  $R^2=0.94$ , RMSE=5.6ms
  - ✧ Accuracy  $R^2=0.82$ , RMSE=6%

# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ✧ Latency  $R^2=0.94$ , RMSE=5.6ms
  - ✧ Accuracy  $R^2=0.82$ , RMSE=6%



# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ✧ Latency  $R^2=0.94$ , RMSE=5.6ms
  - ✧ Accuracy  $R^2=0.82$ , RMSE=6%

# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
- ✧ Latency  $R^2=0.94$ , RMSE=5.6ms
- ✧ Accuracy  $R^2=0.82$ , RMSE=6%
- ✧ Anderson & Reder (1999)



# Model Fit

---

- ✧ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ✧ Latency  $R^2=0.94$ , RMSE=5.6ms
  - ✧ Accuracy  $R^2=0.82$ , RMSE=6%
- ✧ Anderson & Reder (1999)
  - ✧ Full model

# Model Fit

---

- \* Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - \* Latency  $R^2=0.94$ , RMSE=5.6ms
  - \* Accuracy  $R^2=0.82$ , RMSE=6%
- \* Anderson & Reder (1999)
  - \* Full model
    - \* Perception / Action



# Model Fit

---

- ❖ Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - ❖ Latency  $R^2=0.94$ , RMSE=5.6ms
  - ❖ Accuracy  $R^2=0.82$ , RMSE=6%
- ❖ Anderson & Reder (1999)
  - ❖ Full model
    - ❖ Perception / Action
    - ❖ Study-phase

# Model Fit

---

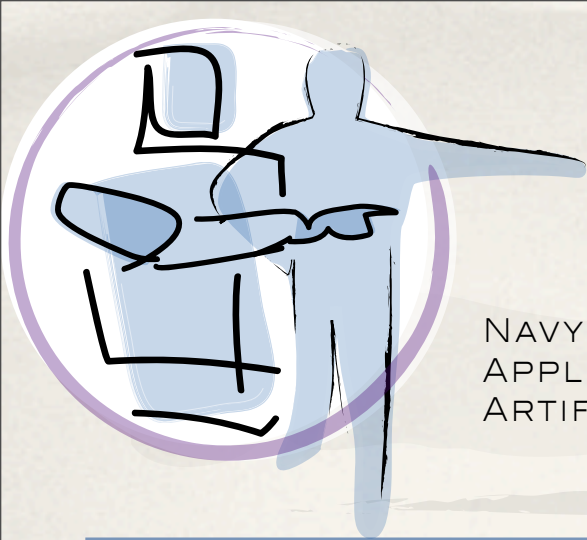
- \* Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - \* Latency  $R^2=0.94$ , RMSE=5.6ms
  - \* Accuracy  $R^2=0.82$ , RMSE=6%
- \* Anderson & Reder (1999)
  - \* Full model
    - \* Perception / Action
    - \* Study-phase
    - \* Drop-out testing



# Model Fit

---

- \* Tucker & Ellis (1998) (Harrison & Trafton (2010) )
  - \* Latency  $R^2=0.94$ , RMSE=5.6ms
  - \* Accuracy  $R^2=0.82$ , RMSE=6%
- \* Anderson & Reder (1999)
  - \* Full model
    - \* Perception / Action
    - \* Study-phase
    - \* Drop-out testing
    - \* Testing



NAVY CENTER FOR  
APPLIED RESEARCH IN  
ARTIFICIAL INTELLIGENCE



# Questions?

# Insights?

# Ideas?